Compilers and Automata Theory

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  - Produces efficient code.

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- Problem: Can one method of "writing down instructions" be more powerful than another?

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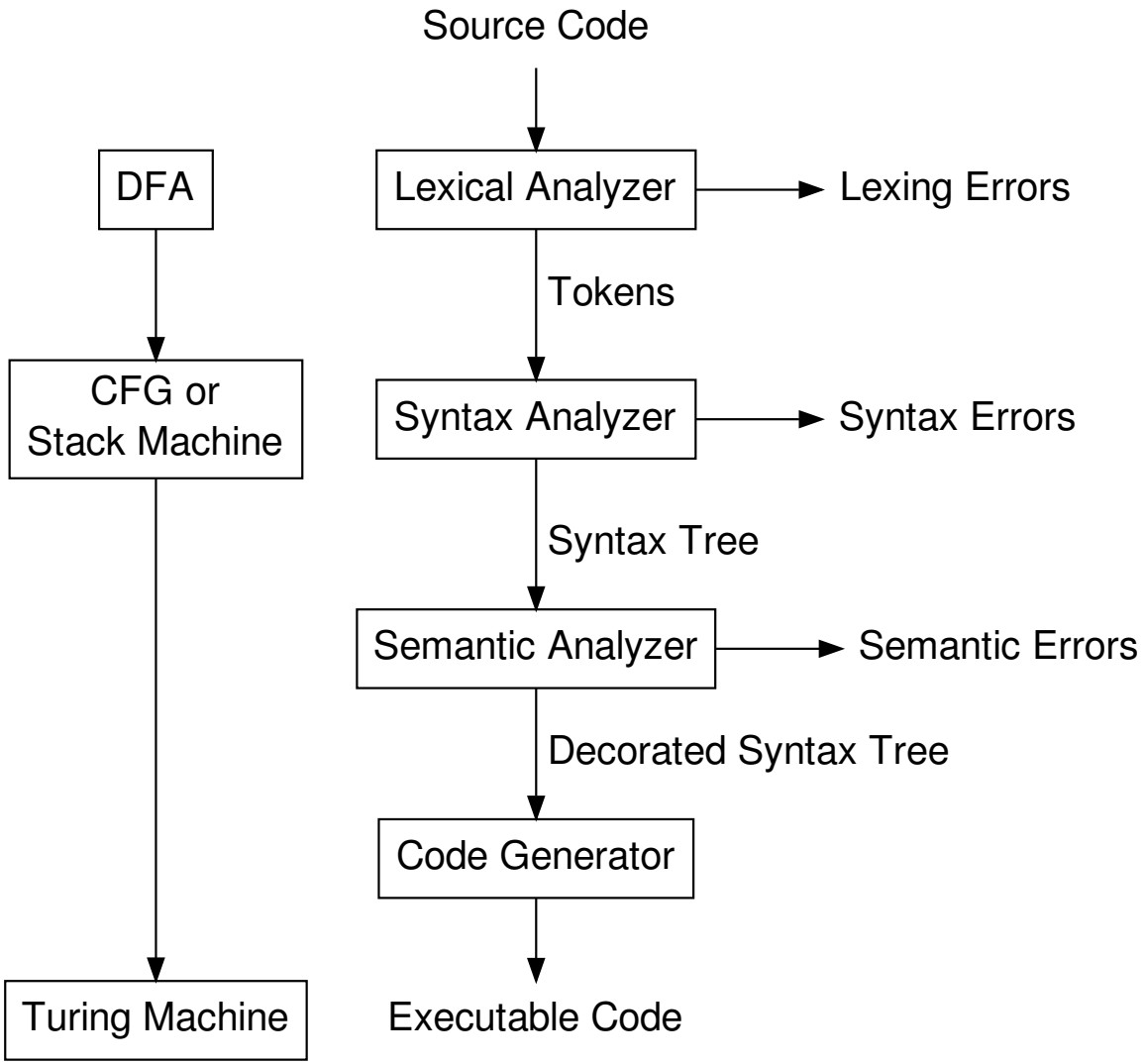
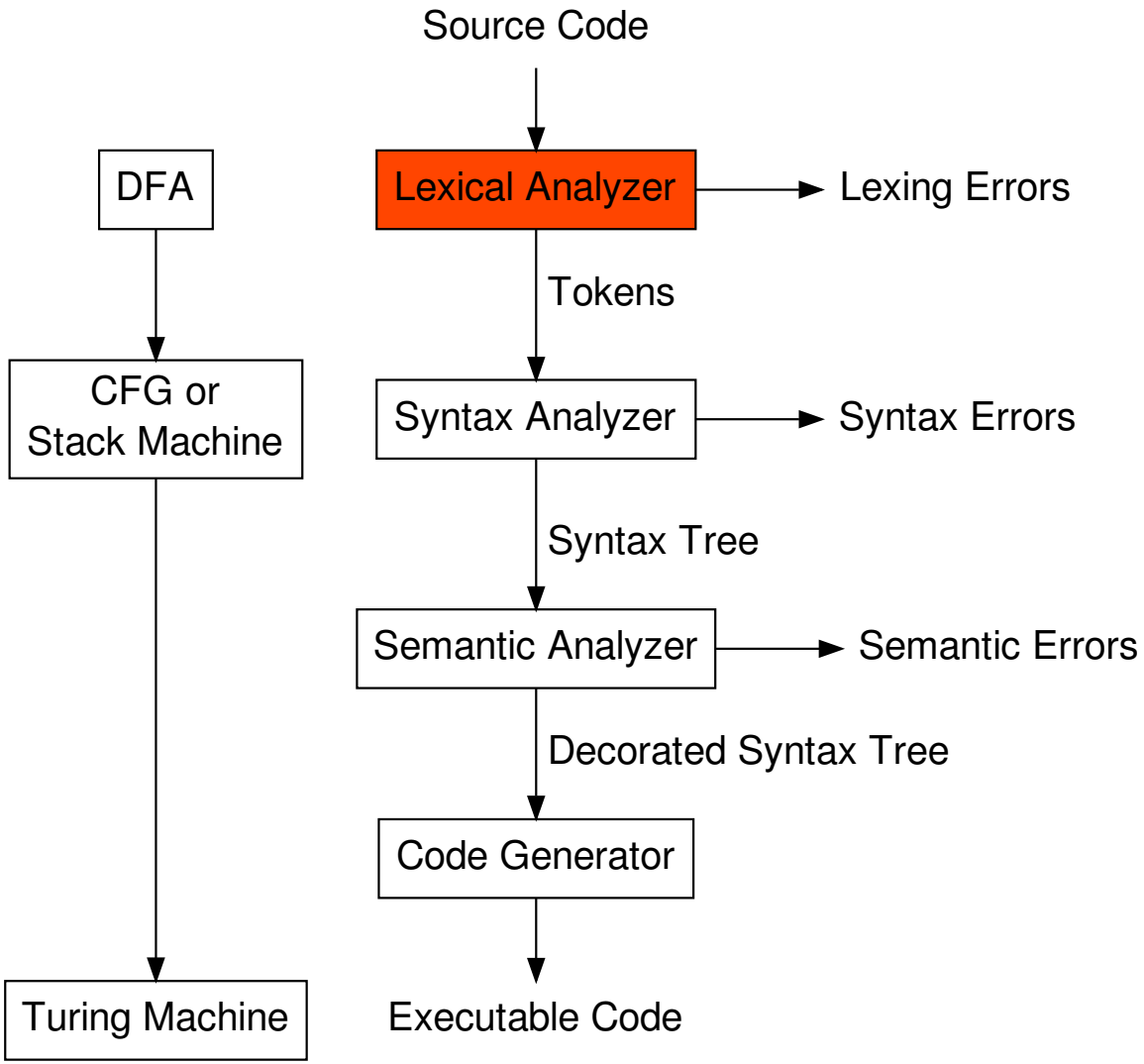
- ...when they succeed in producing output.
- ...when they give you error messages.

Phases of Compilation



Lexical Analysis



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- Recognize numerical literals: `453` $\mapsto$ `INT_LIT(453)`.
- Recognize "spelling" errors:
  - `3.14.14` doesn't define a valid floating point number.
  - `314!!foo` is not a valid variable name in C.

Example

A line of C code (30 characters with whitespace):

```
int foo = bar + 453;
```

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A line of C code (30 characters with whitespace):

```
int foo = bar + 453;
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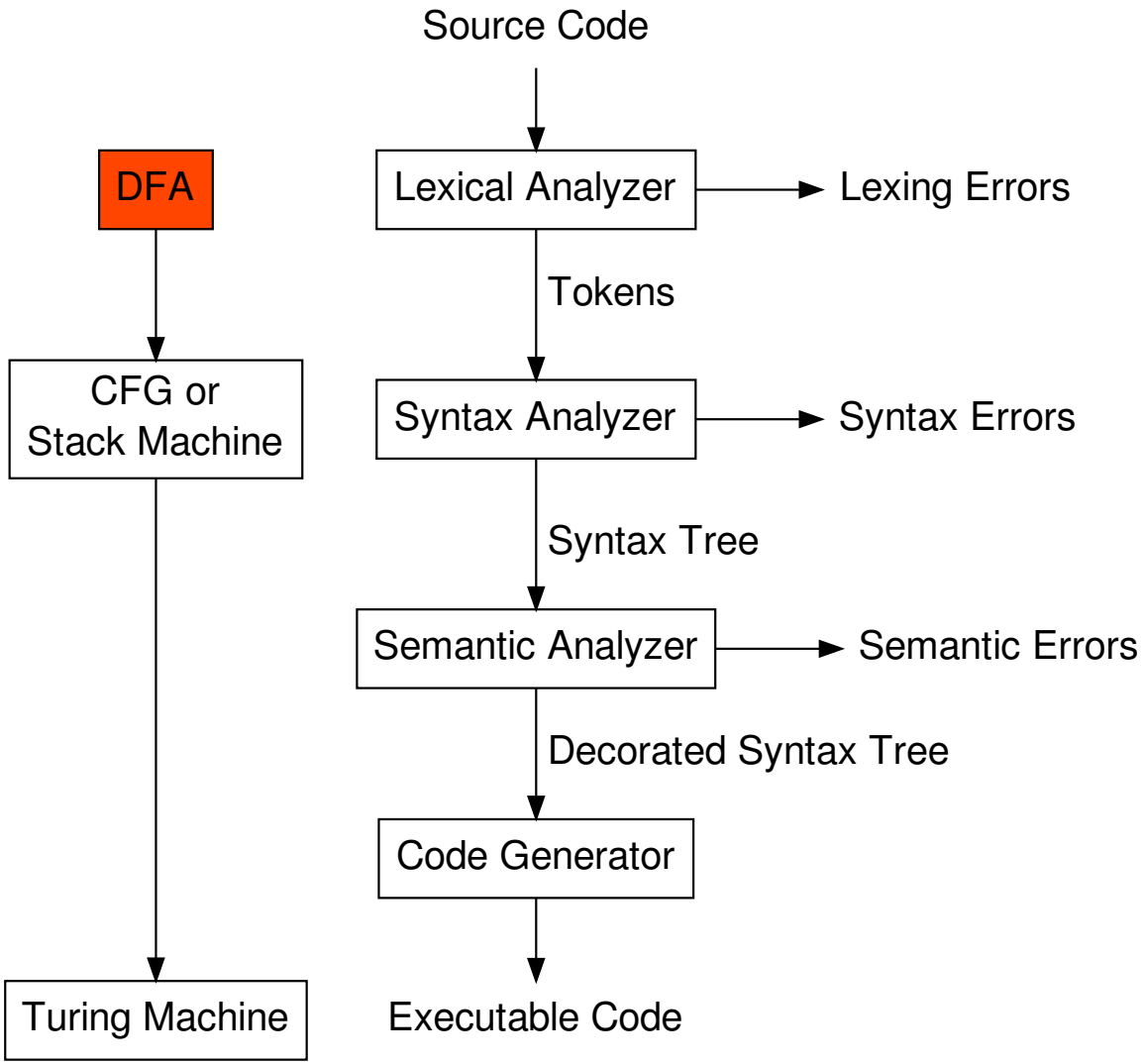
becomes a sequence of 7 tokens:

```
IDENT(int), IDENT(foo), EQUALS, IDENT(bar), PLUS,
INTLIT(453), SEMICOLON
```

Deterministic Finite Automatons (DFAs)



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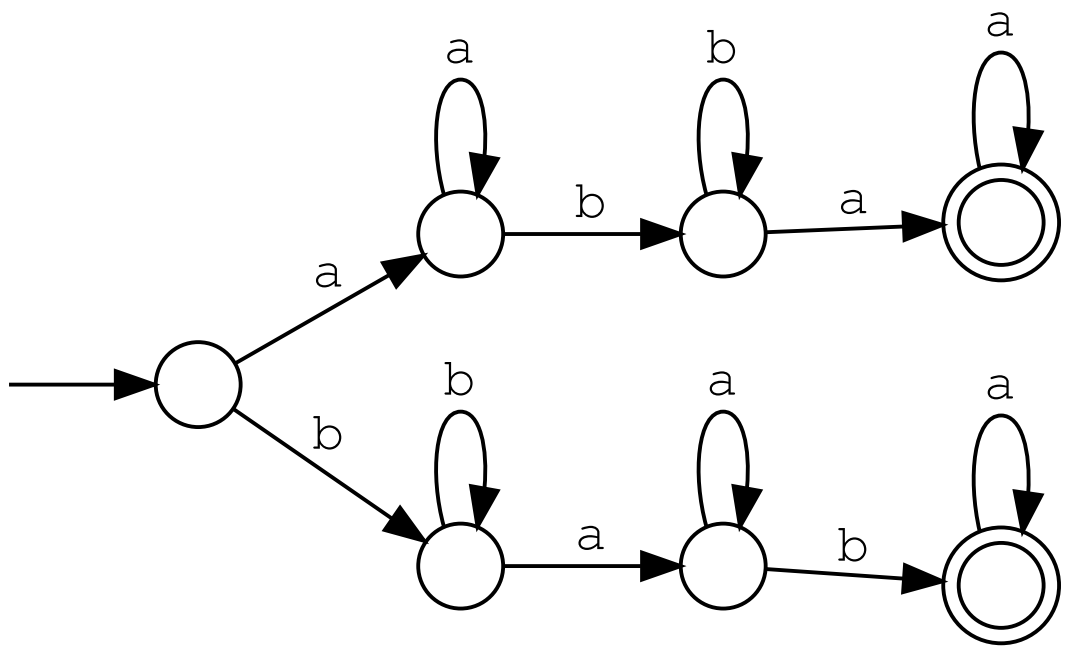

DFAs are decorated directed graphs.

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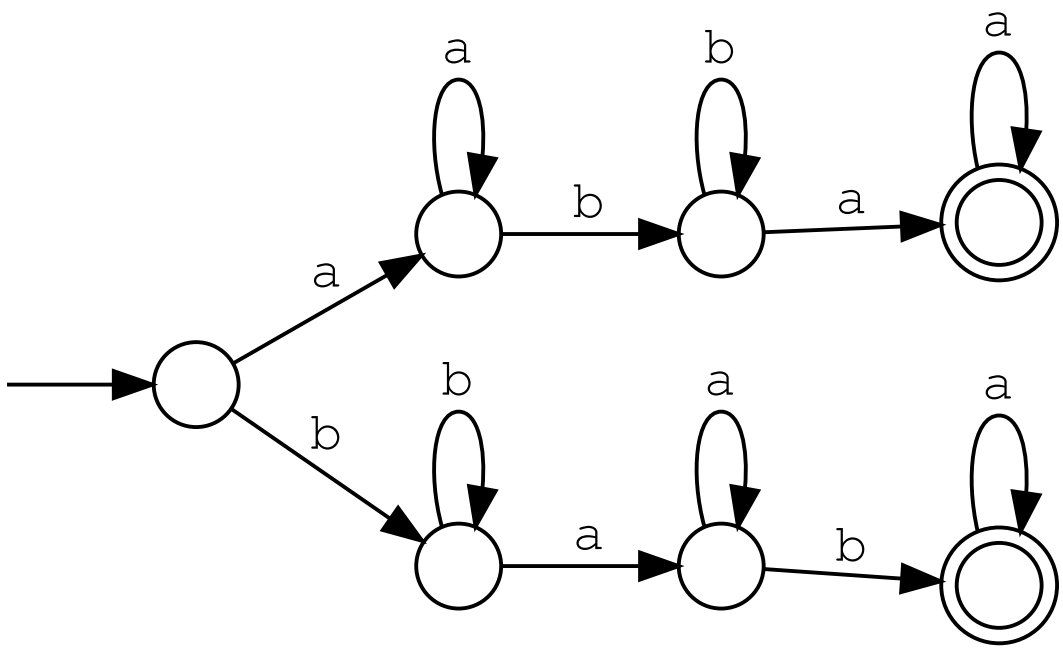

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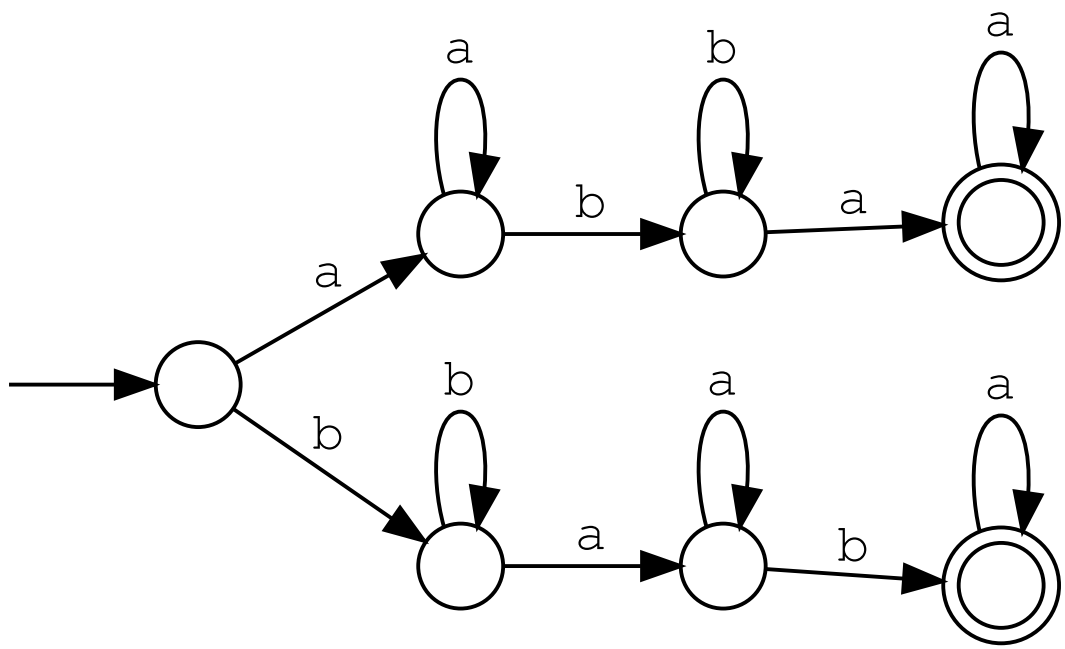

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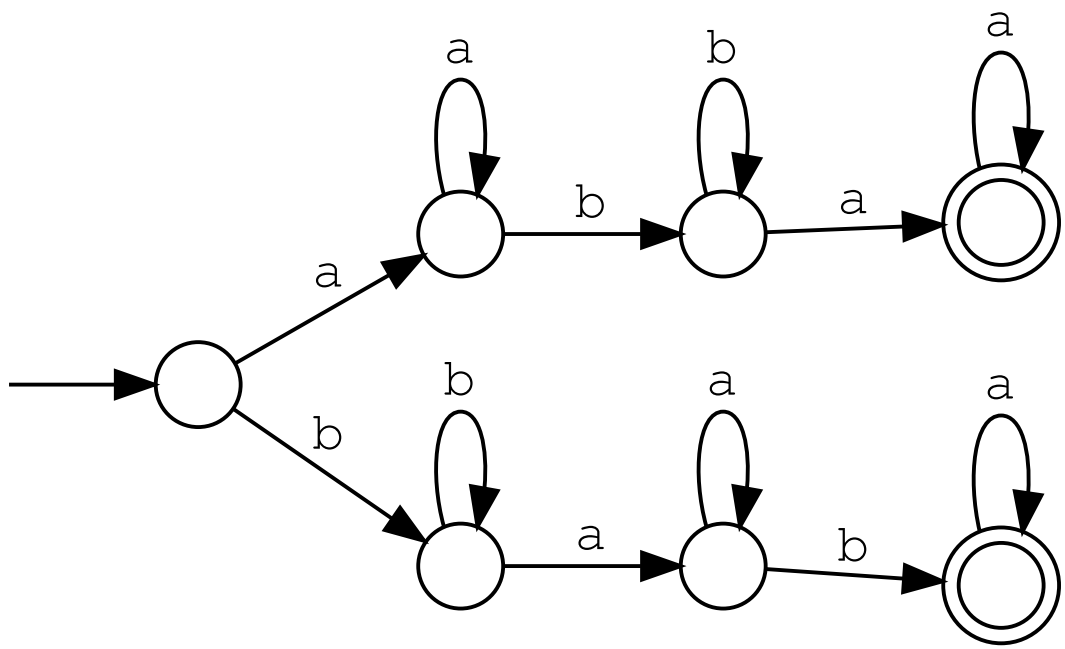

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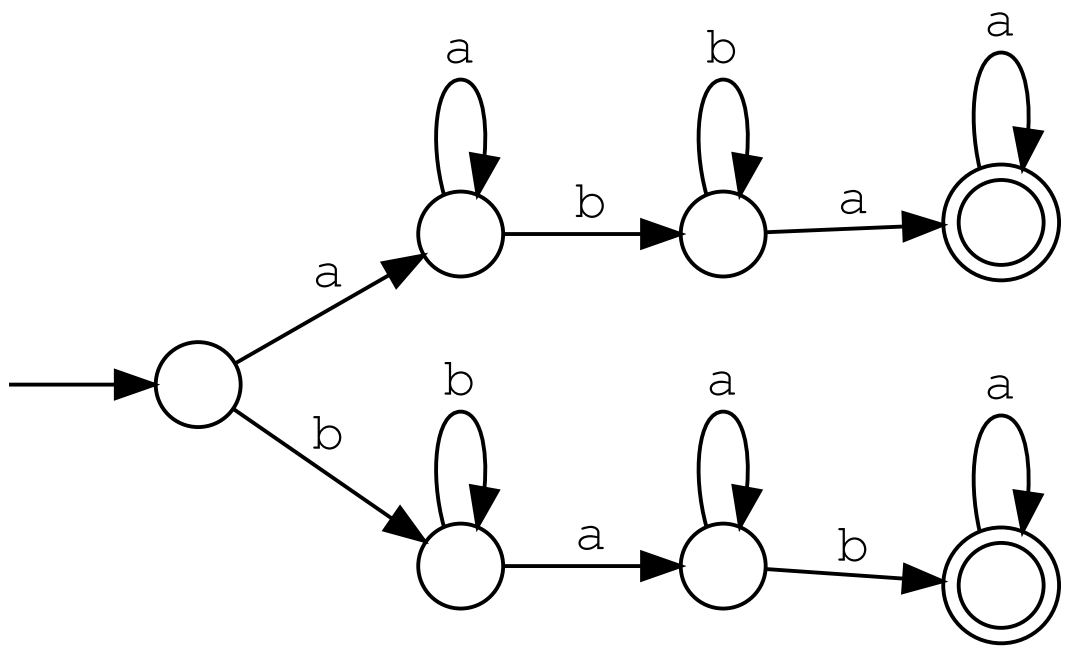

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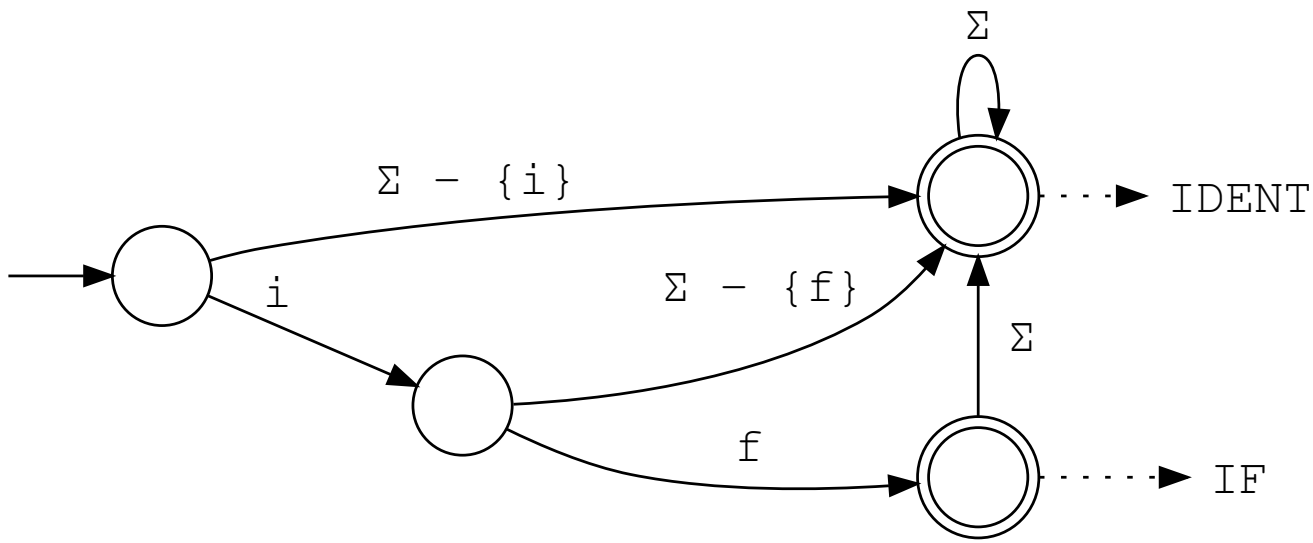
A string is **recognized** (or **accepted**) by a given DFA if it describes a path from the start node to an end node.

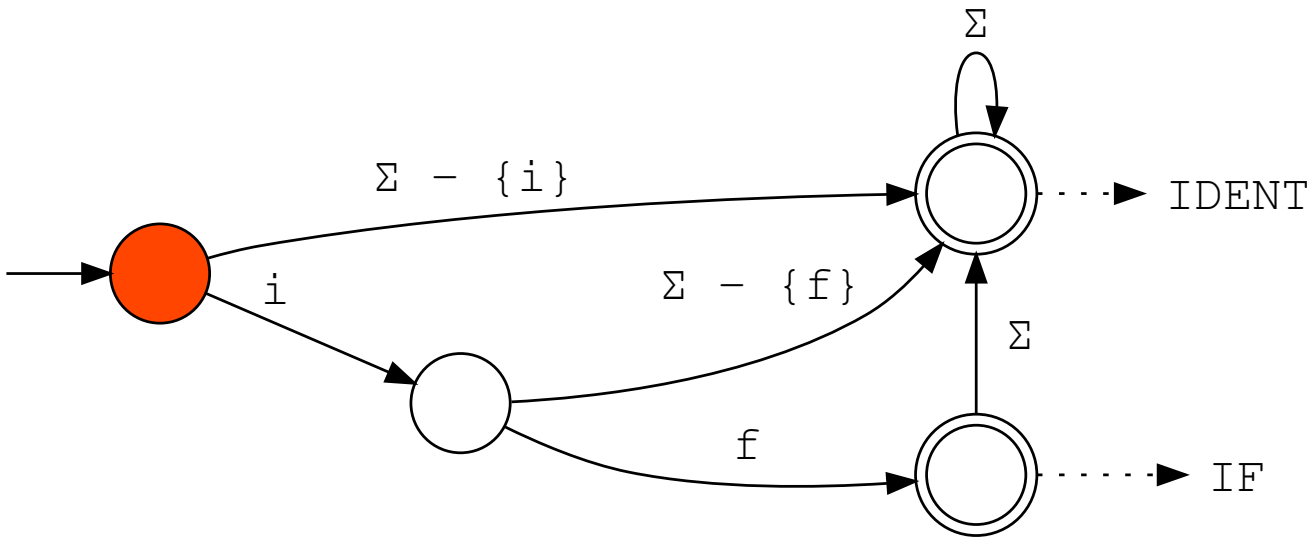
DFA Example

A DFA that recognizes identifiers (variable names) and the `if` token.



DFA Example

Input: `if iota`



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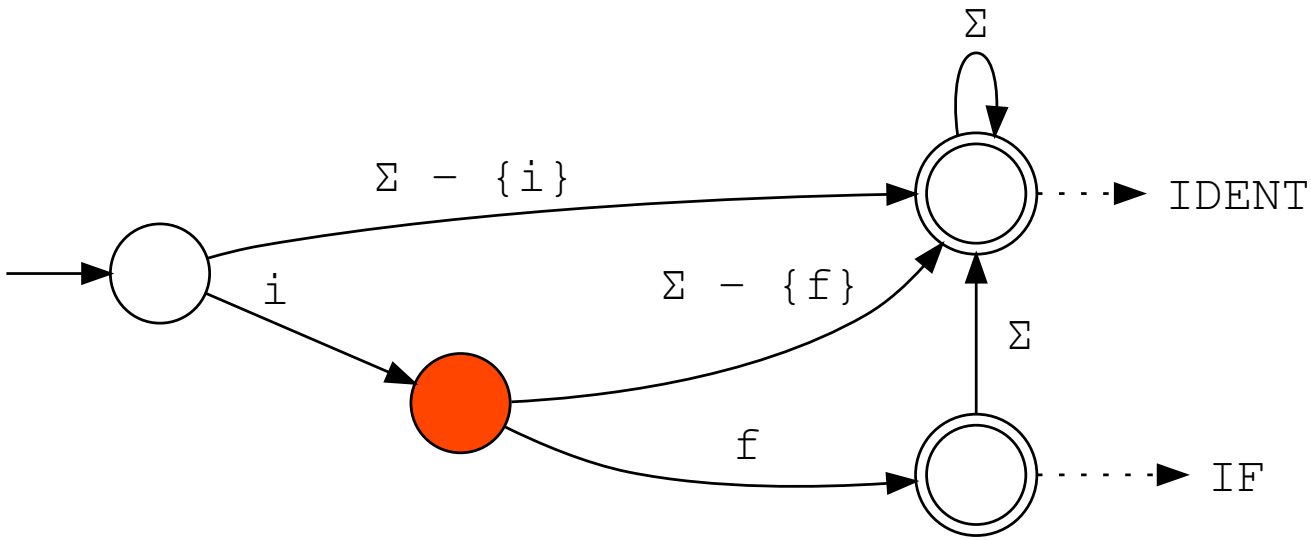

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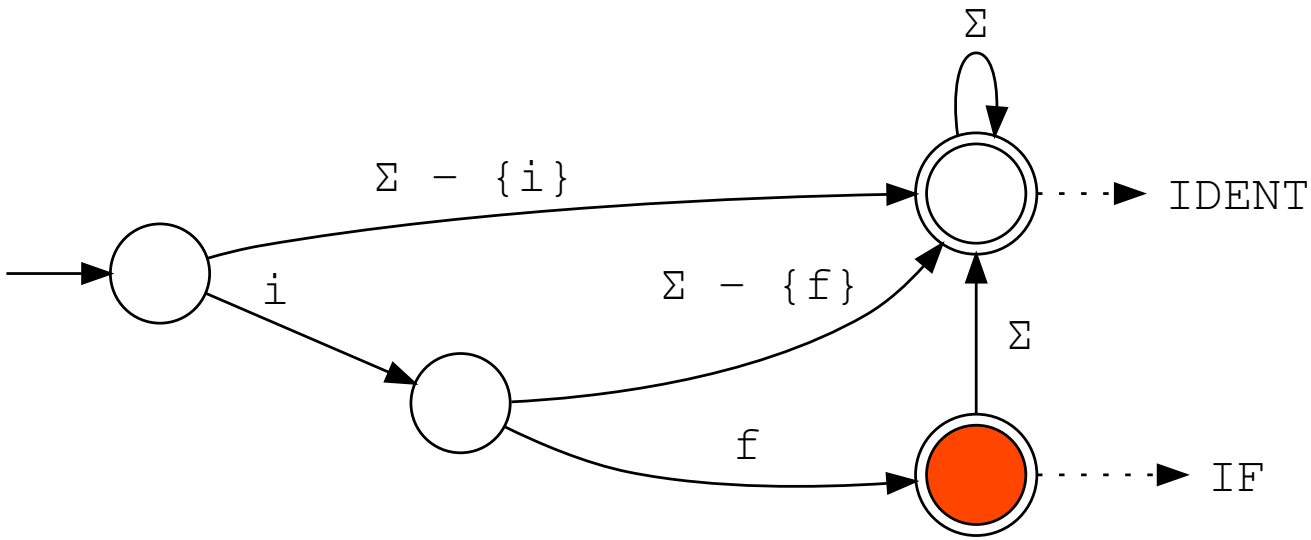

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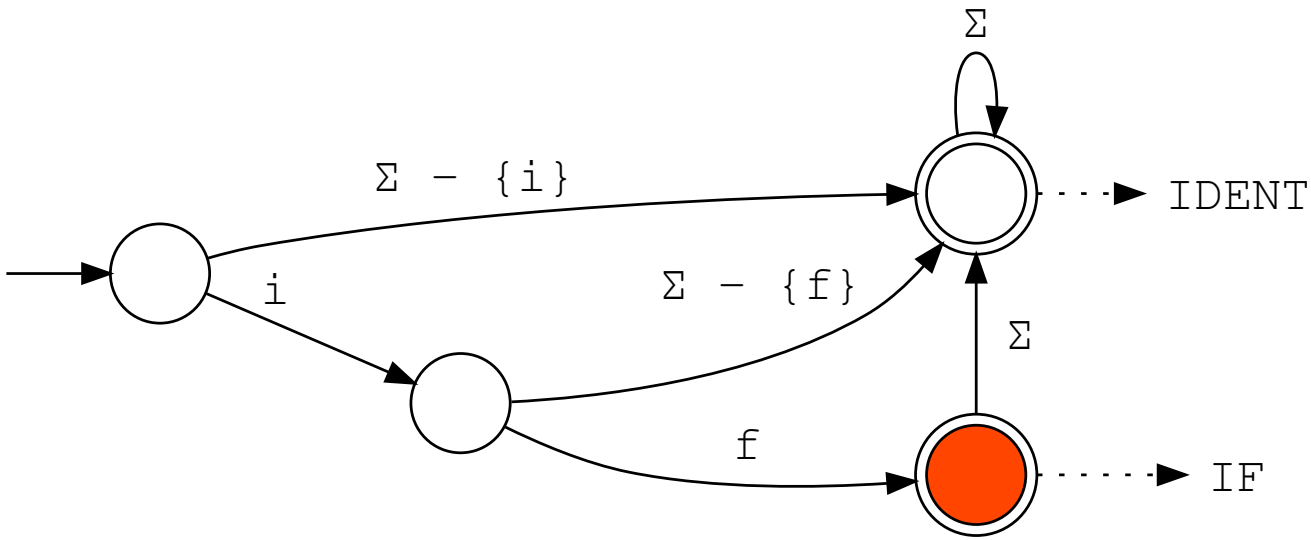

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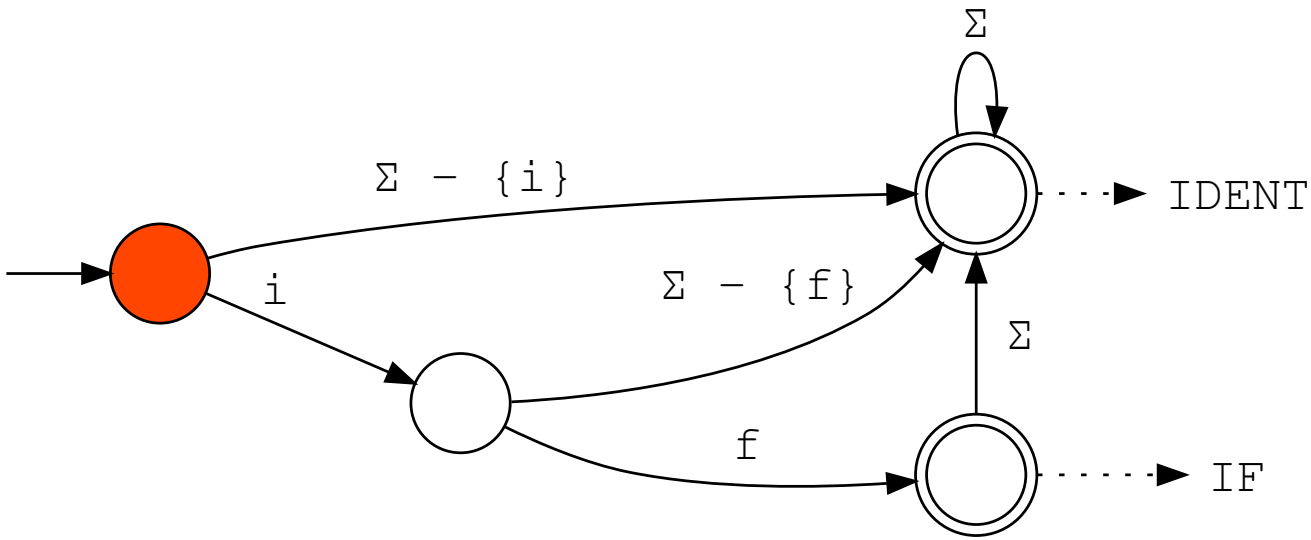

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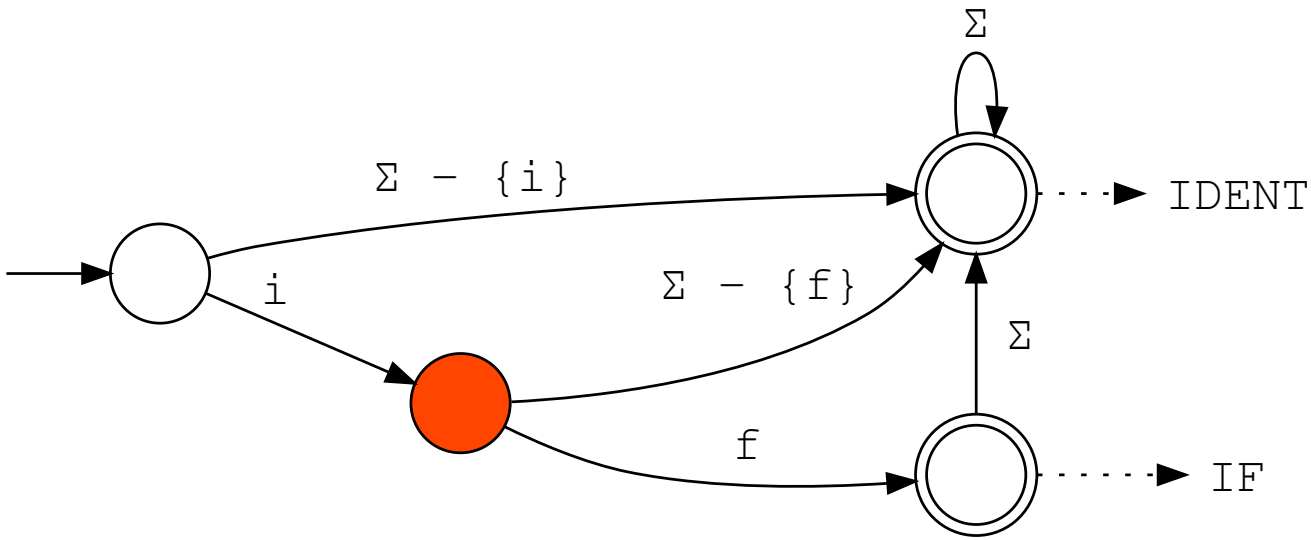

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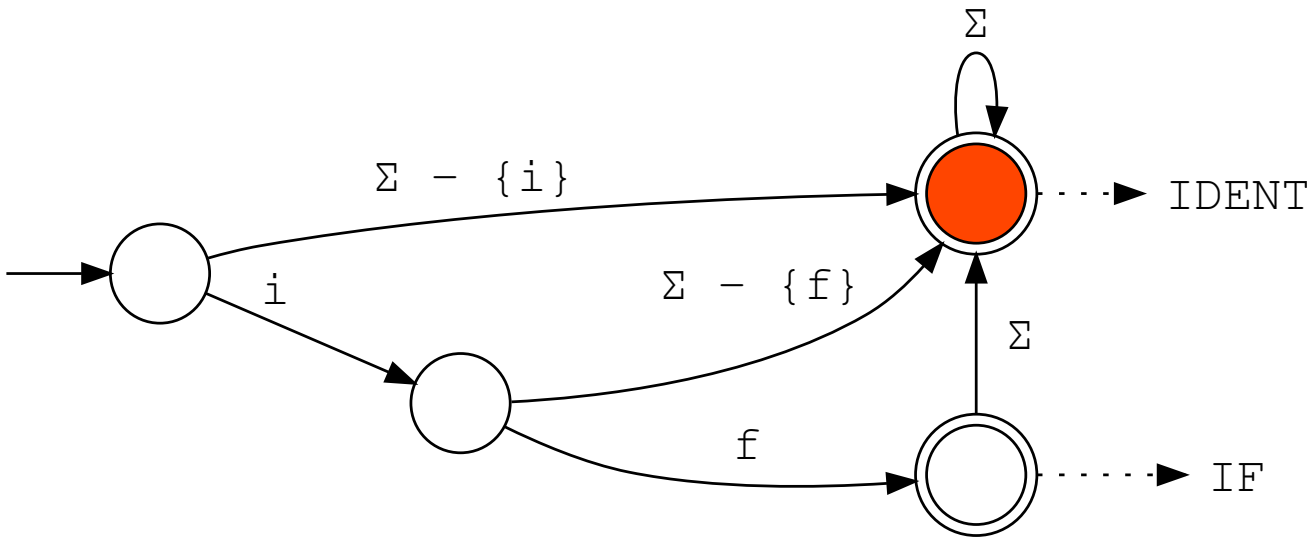

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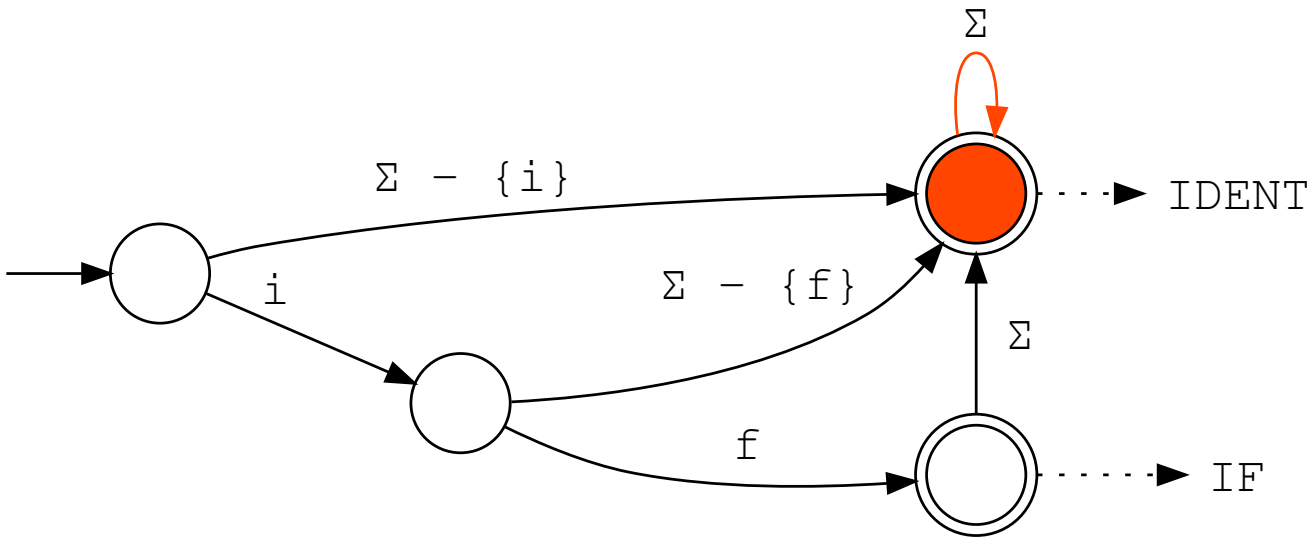

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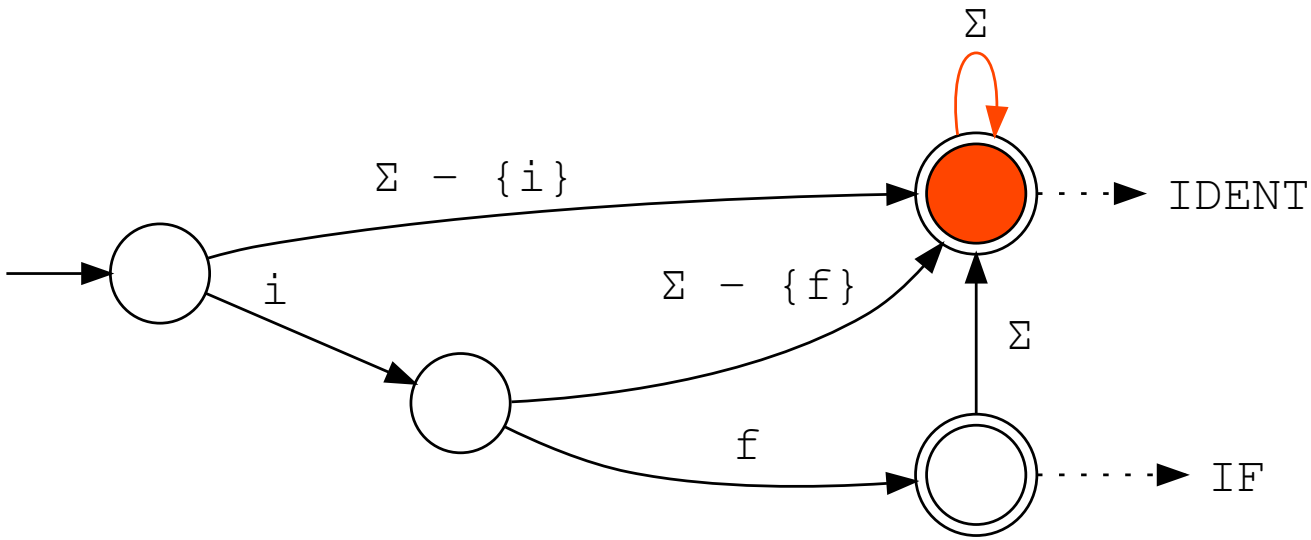
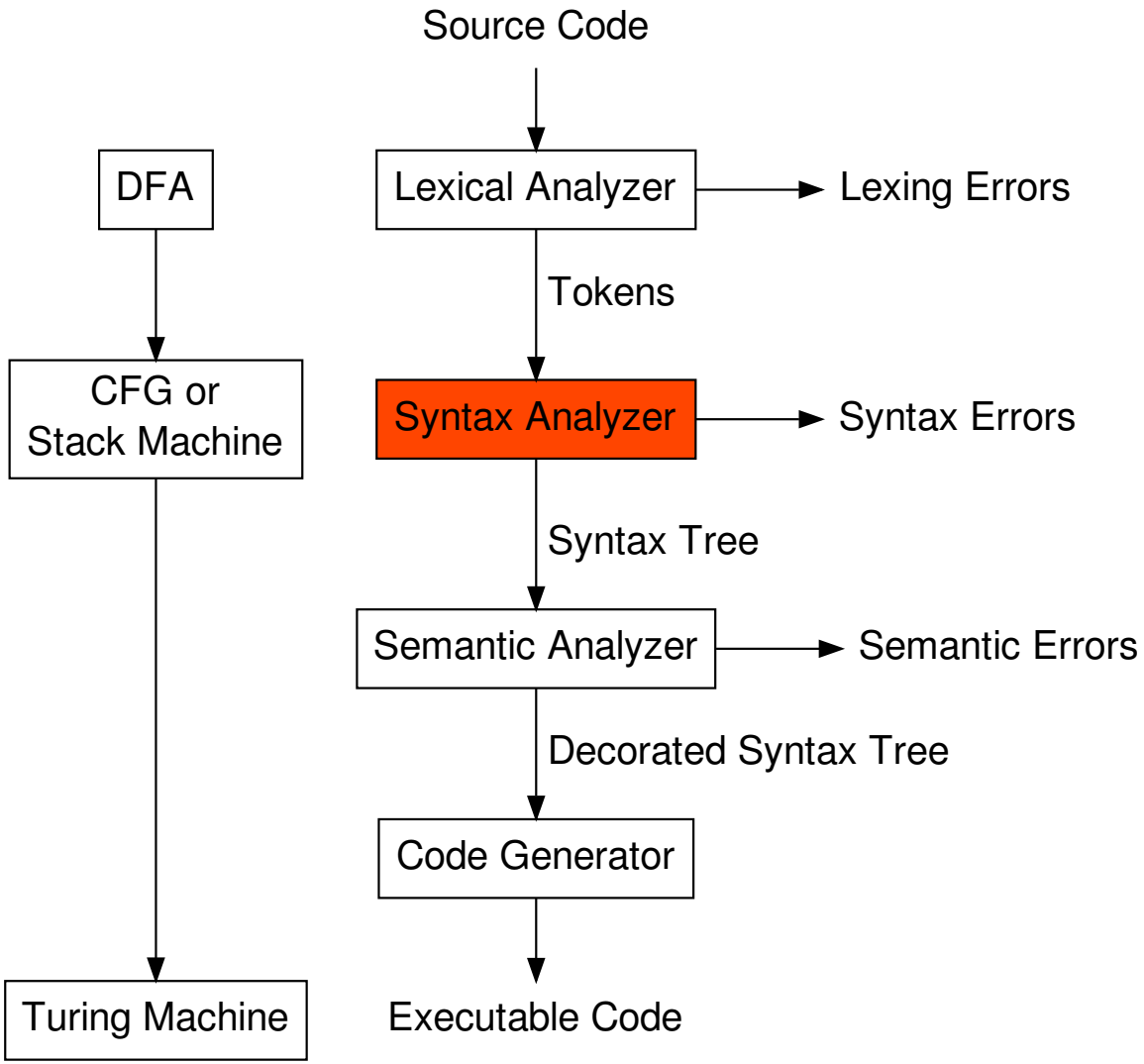


Output: `IF, IDENT(iota),`

Syntax Analysis



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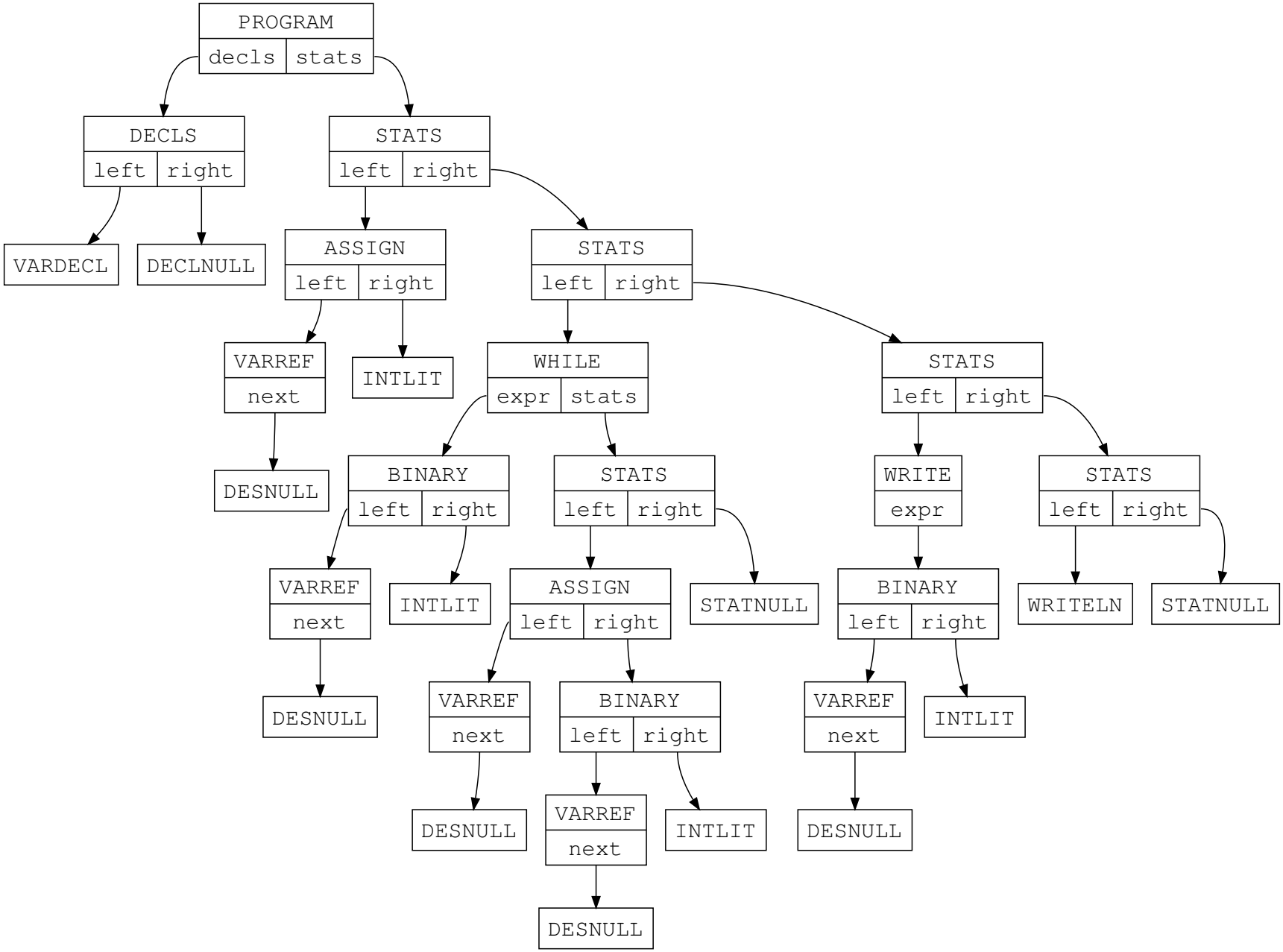
- The tree encodes the grammatical structure of the program.
- We'll store further information about the program on the tree.
- Syntax trees are **very** convenient for operating on programs.

Example:

```
PROGRAM P;

VAR i : INTEGER;
BEGIN
    i := 1;
    WHILE i < 10 DO
        i := i + 1;
    ENDDO;
    WRITE i - 10; WRITELN;
END.
```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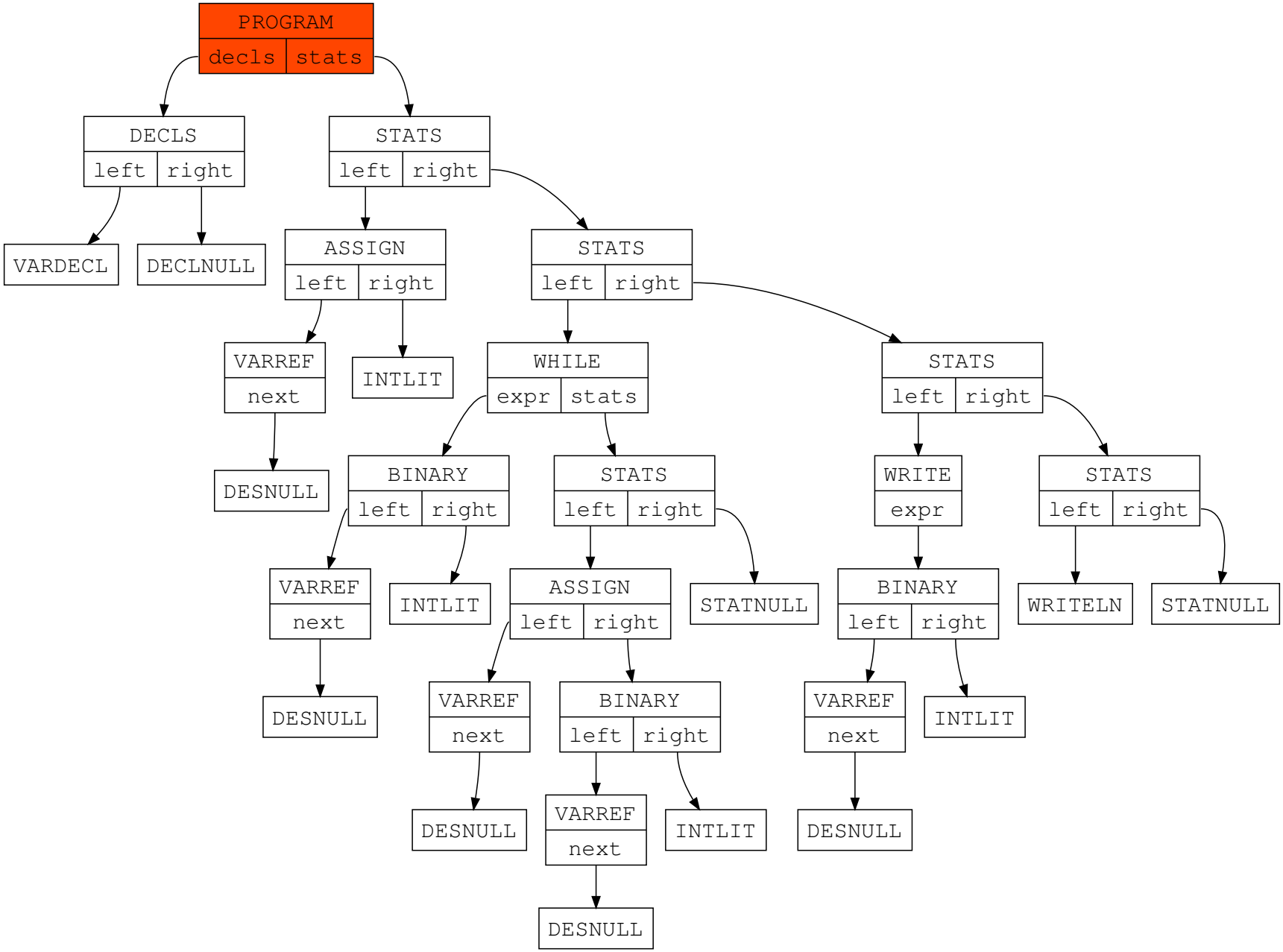
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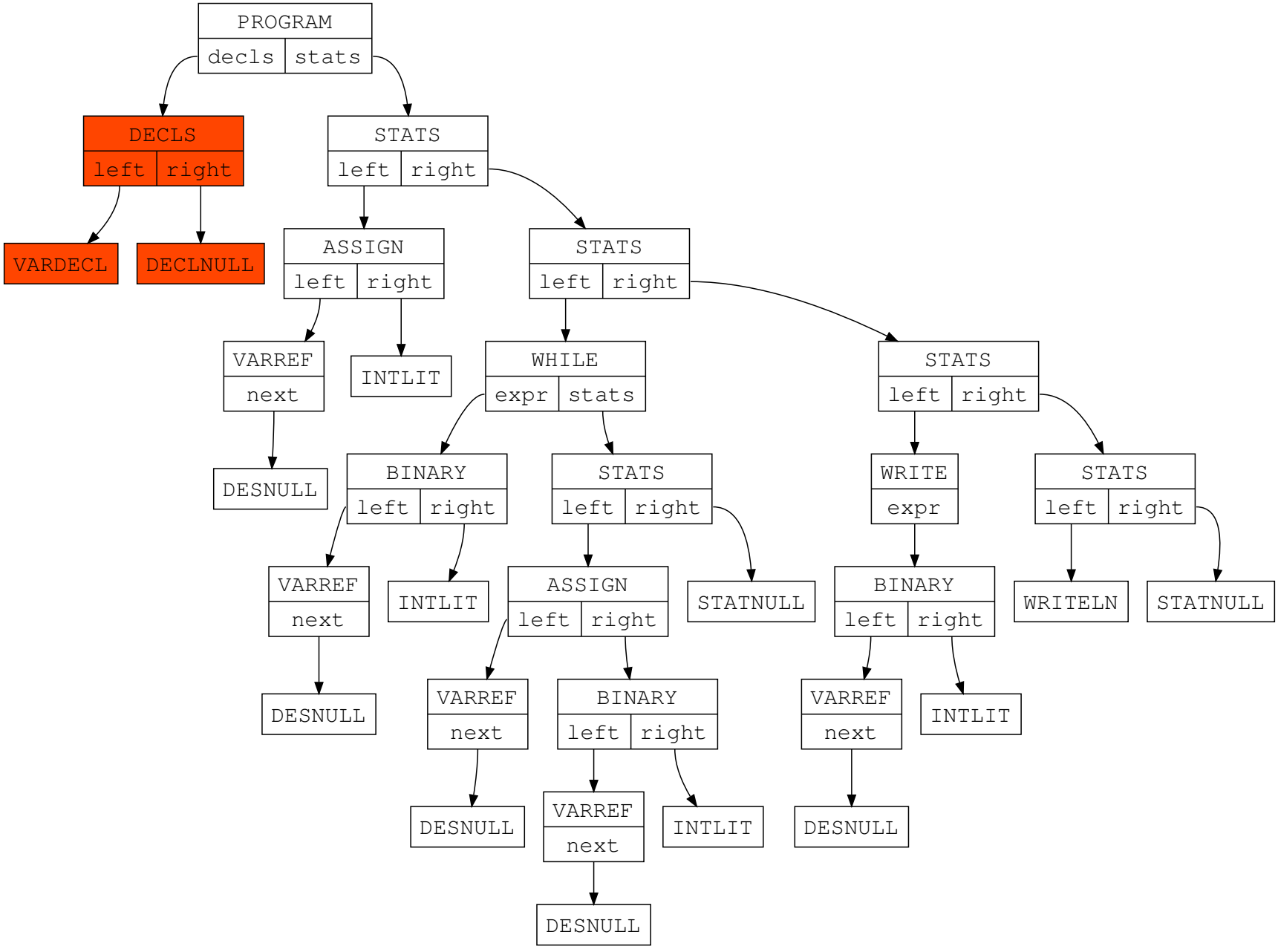
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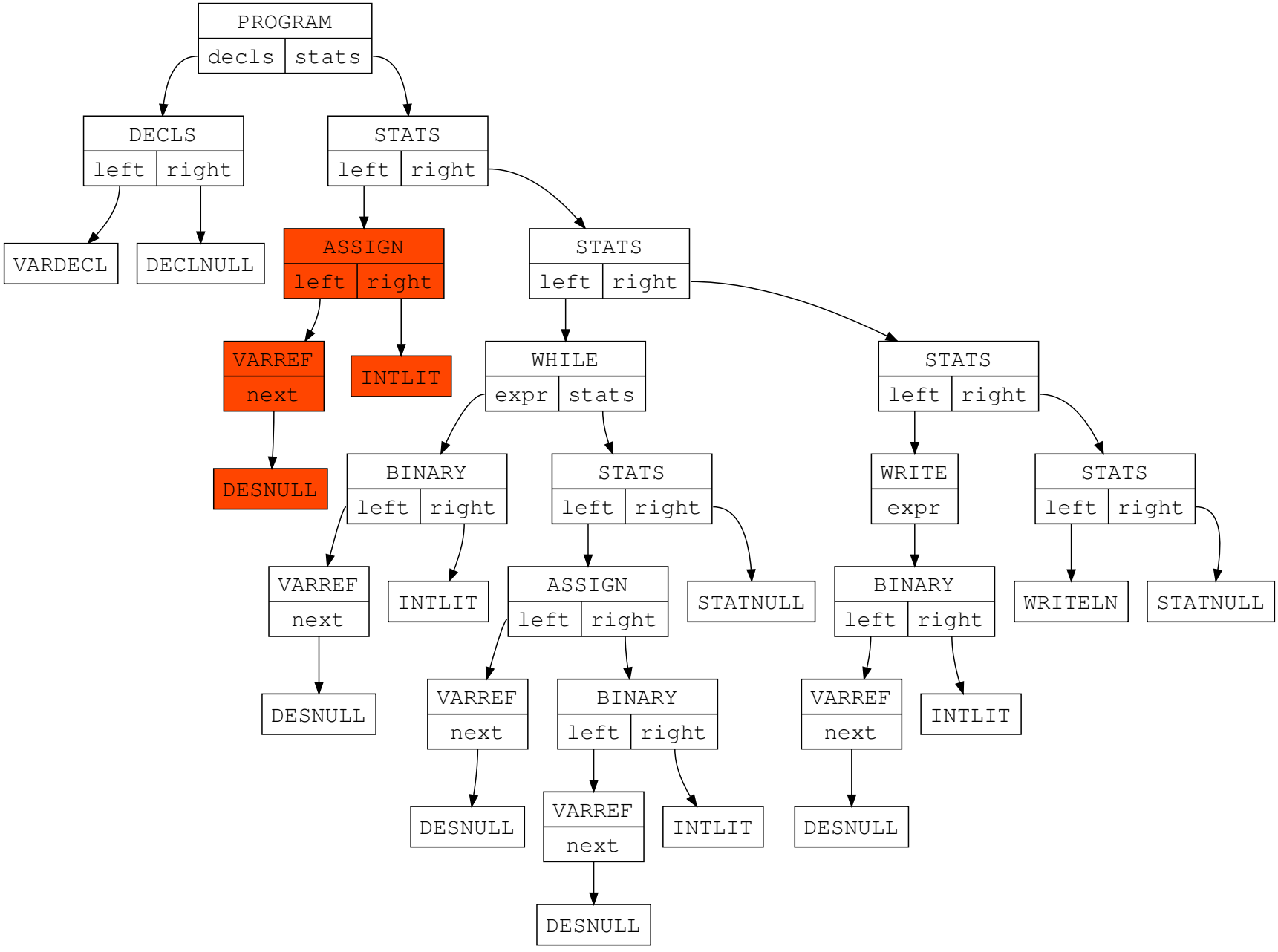
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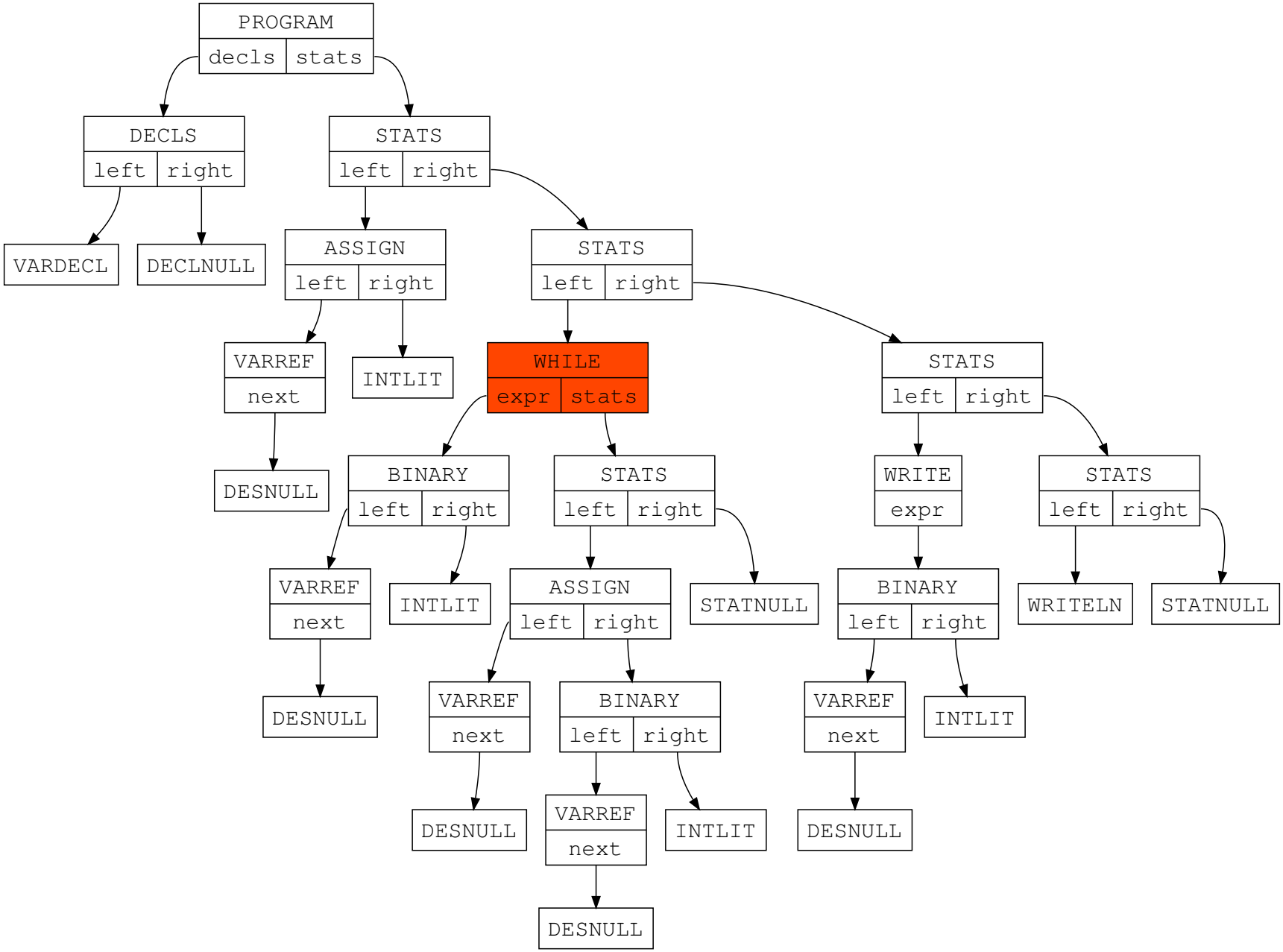
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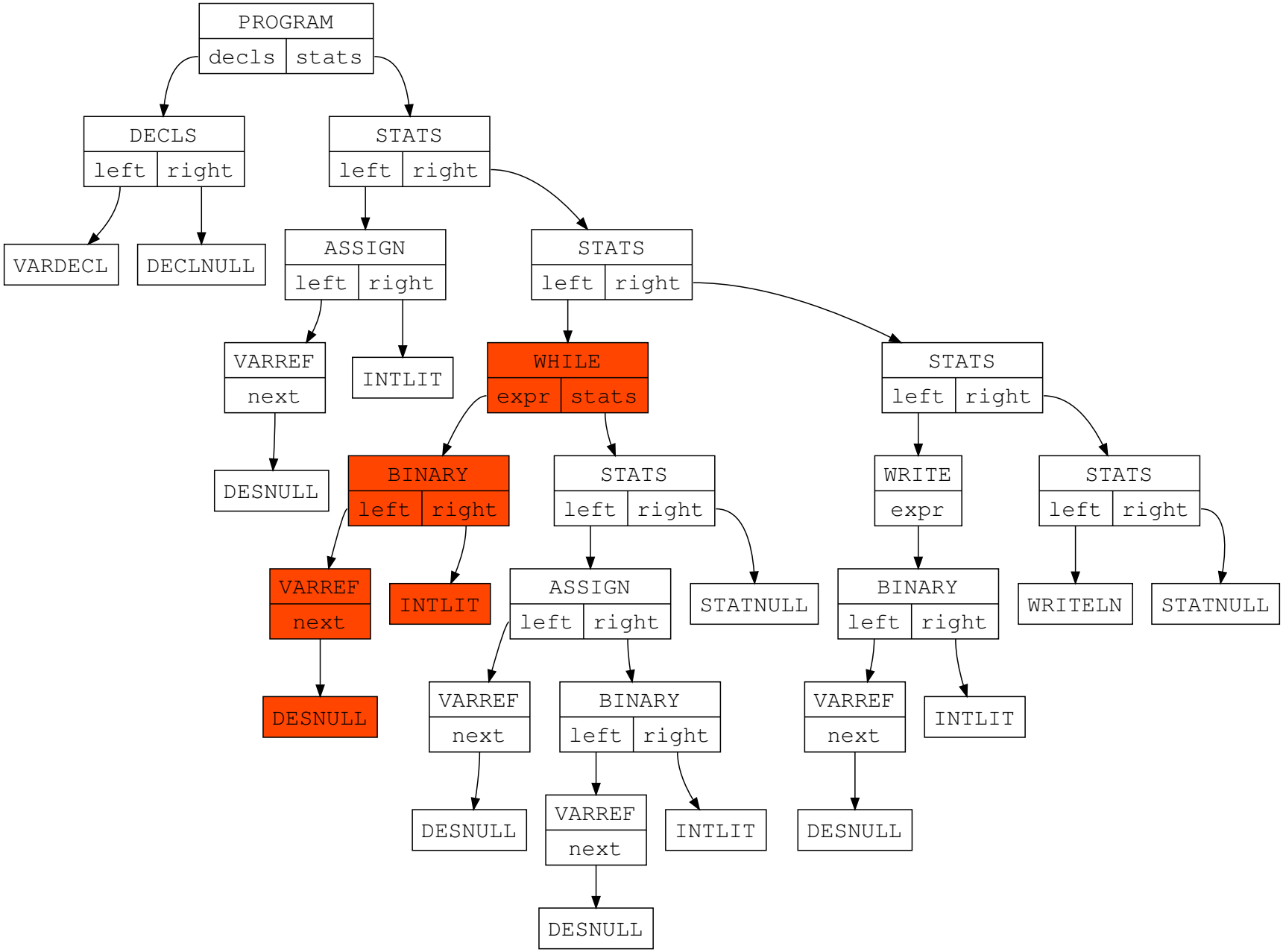
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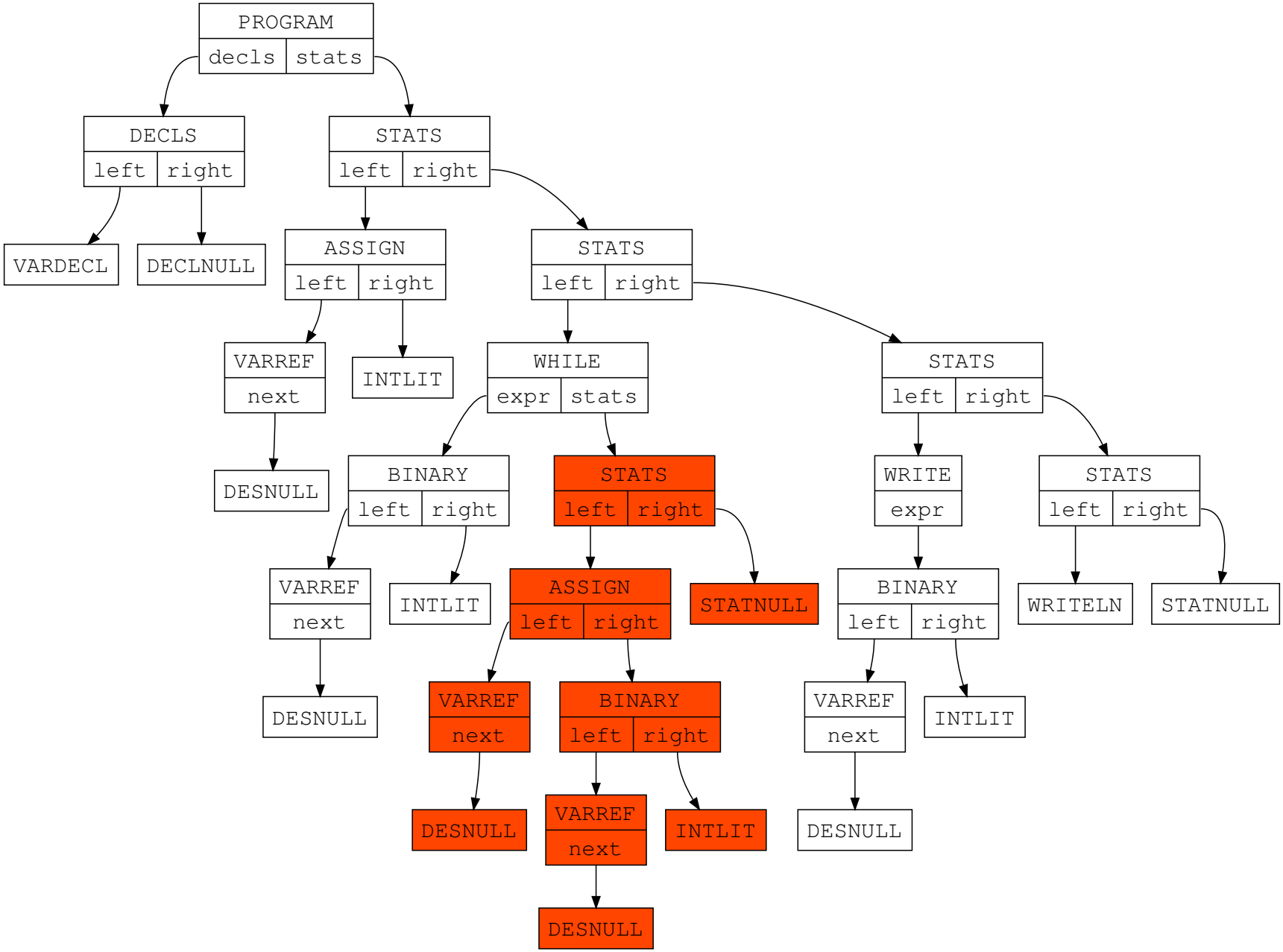
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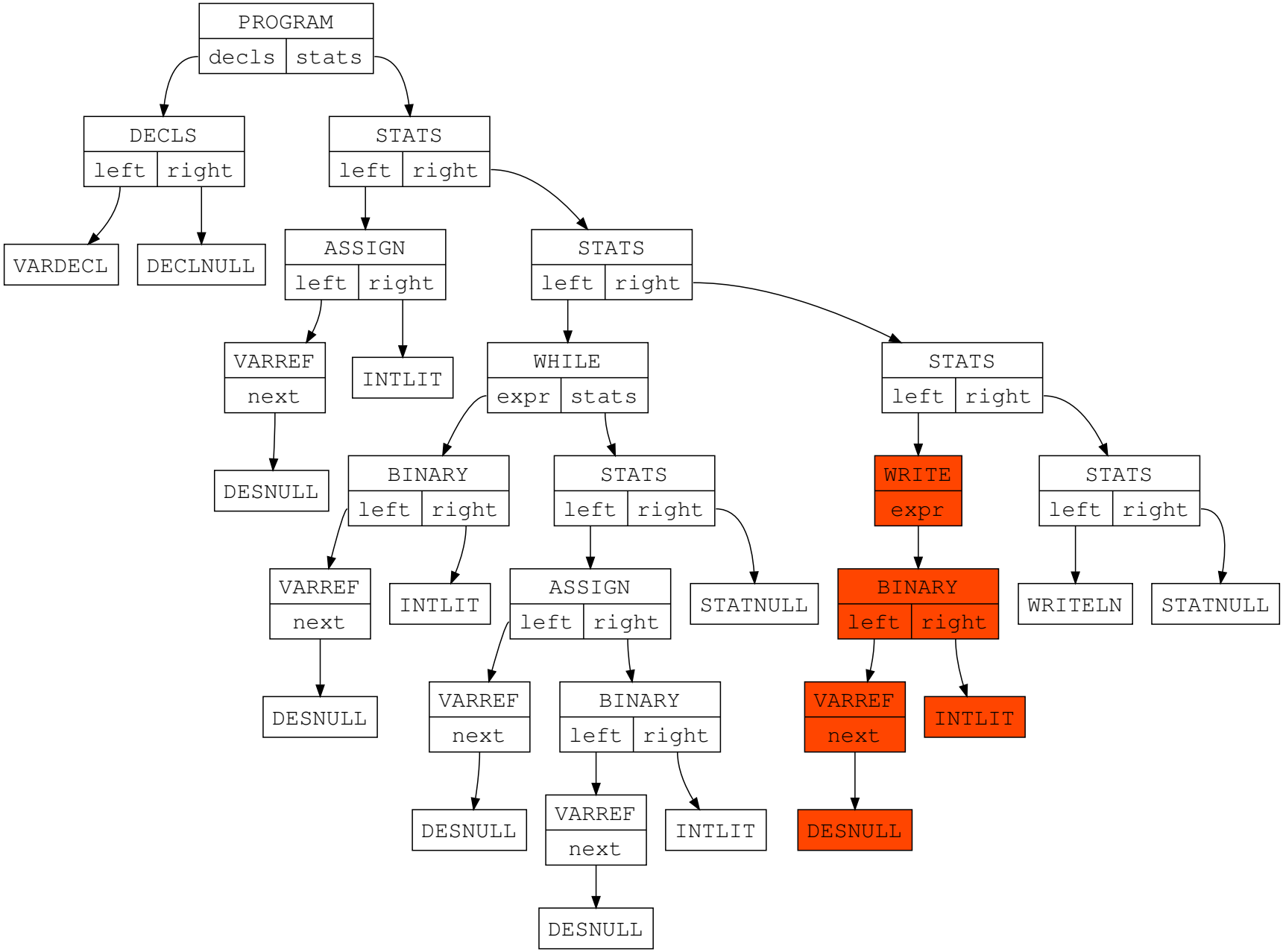
INTLIT
ASSIGN
left
right
STATNULL
BINARY
left
right
WRITELN
STATNULL
DESNULL
VARREF
next
BINARY
left
right
VARREF
next
INTLIT
DESNULL
VARREF
next
INTLIT
DESNULL
DESNULL


PROGRAM
decls
stats
DECLS
left
right
STATS
left
right
VARDECL
DECLNULL
ASSIGN
left
right
STATS
left
right
VARREF
next
INTLIT
WHILE
expr
stats
STATS
left
right
DESNULL
BINARY
left
right
STATS
left
right
WRITE
expr
STATS
left
right
VARREF
next
INTLIT
ASSIGN
left
right
STATNULL
BINARY
left
right
WRITELN
STATNULL
DESNULL
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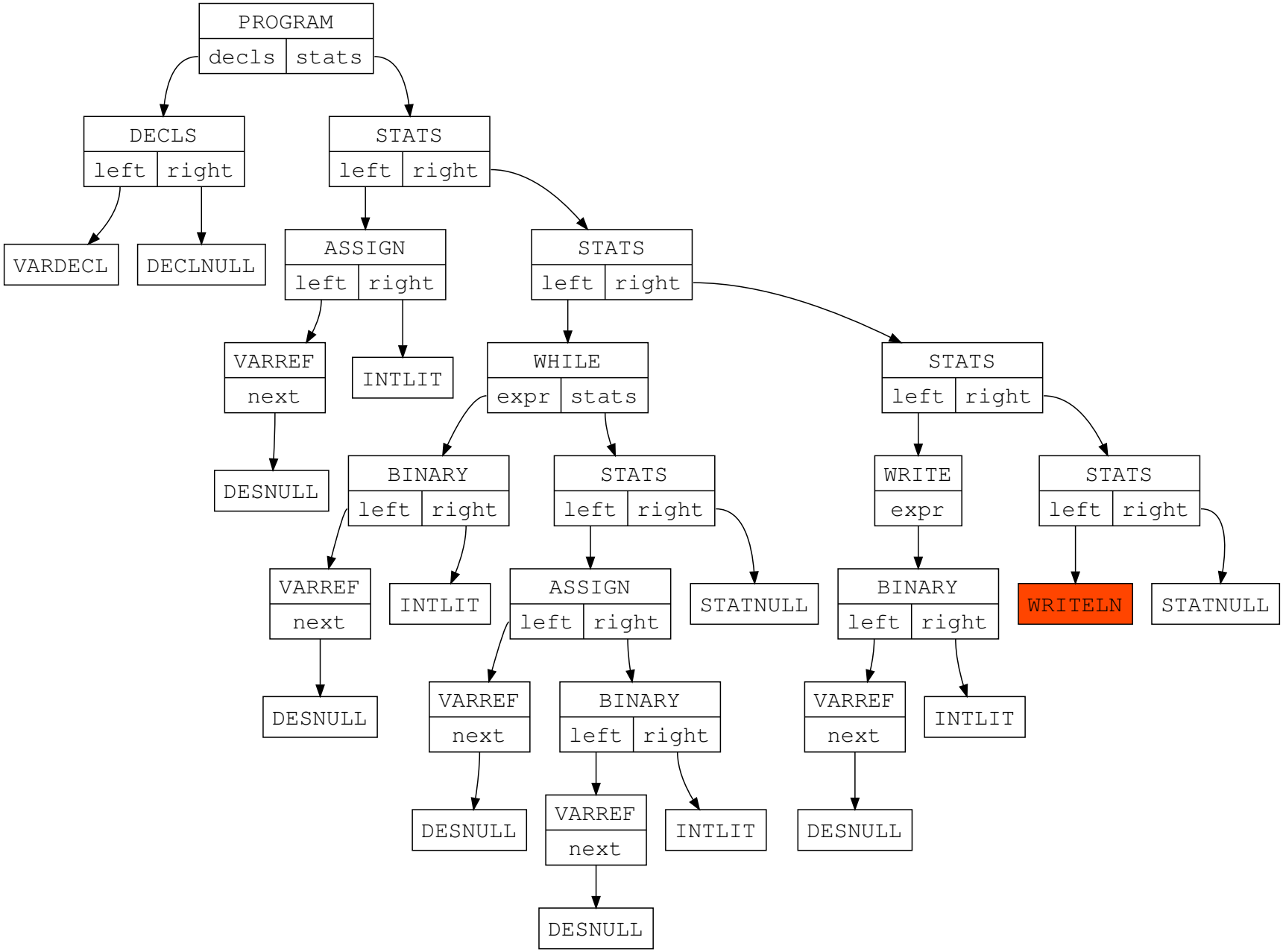
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BINARY
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STATS
left
right
WRITE
expr
STATS
left
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VARREF
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INTLIT
ASSIGN
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right
STATNULL
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right
WRITELN
STATNULL
DESNULL
VARREF
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BINARY
left
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VARREF
next
INTLIT
DESNULL
VARREF
next
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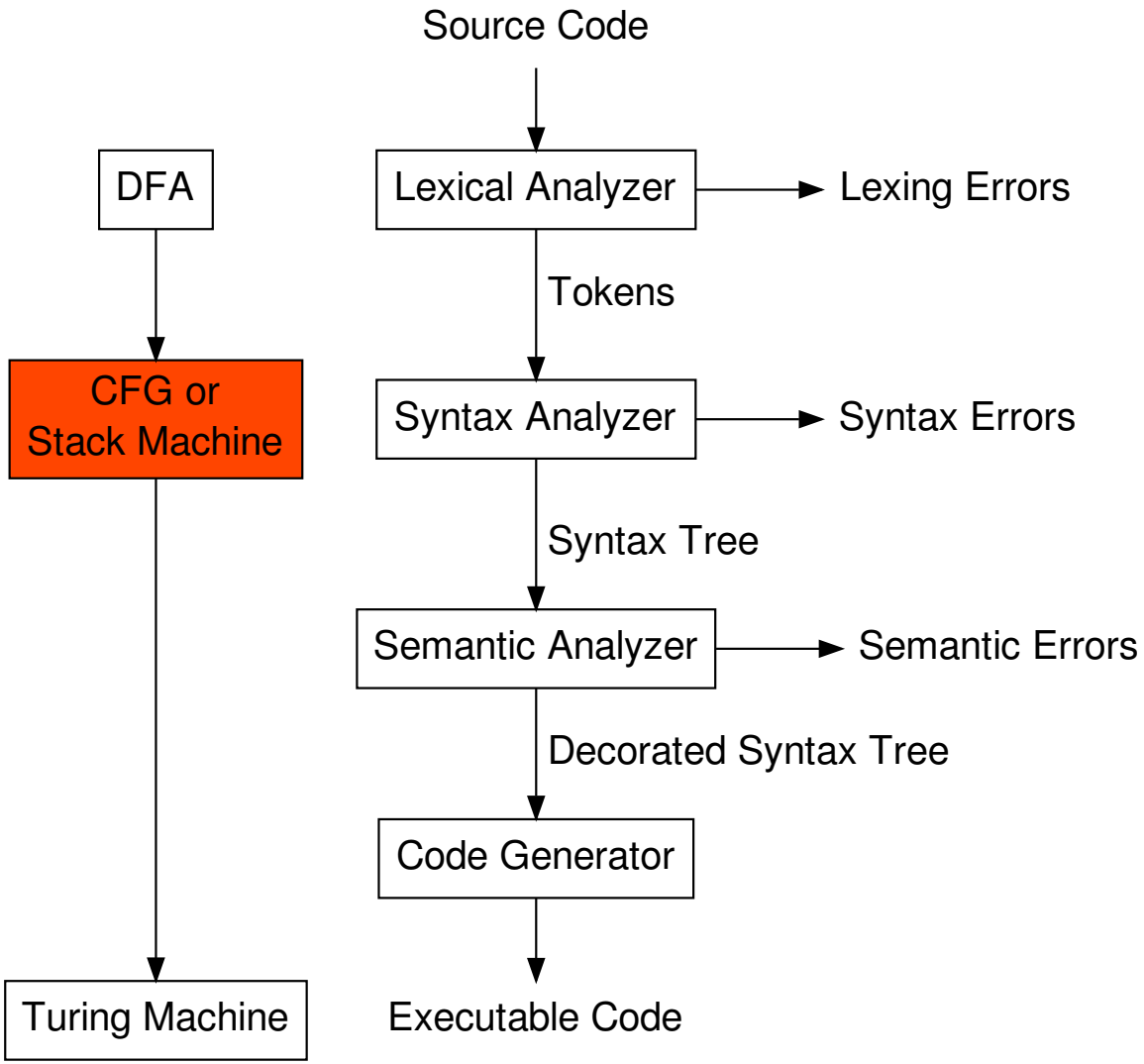
Stack Machines / Context Free Grammars



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Example

Tokens:

$$\Sigma = \{\texttt{while}, \texttt{do}, \texttt{done}, \texttt{true}, \texttt{or}, \texttt{not}, \texttt{:=}, \texttt{;}, \texttt{(}, \texttt{)}, \texttt{x}, \texttt{y}, \texttt{z}\}$$

Grammar:

$$\begin{aligned}
\text{Stats} &\rightarrow \text{Stat}\ \ \texttt{;}\ \ \text{Stats} \mid \varepsilon \\
\text{Stat} &\rightarrow \text{Ident}\ \ \texttt{:=}\ \ \text{Exp} \mid \texttt{while}\ \ \text{Exp}\ \ \texttt{do}\ \ \text{Stats}\ \ \texttt{done} \\
\text{Exp} &\rightarrow \texttt{(}\text{Exp}\ \ \texttt{or}\ \ \text{Exp}\texttt{)} \mid \texttt{(not}\ \ \text{Exp}\texttt{)} \mid \text{Ident} \mid \texttt{true} \\
\text{Ident} &\rightarrow \ \ \texttt{x} \mid \texttt{y} \mid \texttt{z}
\end{aligned}$$

Example Program:

```
while (not x) do x := (not x); done; y := x;
```

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A recipe for creating automatons:

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- Push symbols onto the stack.
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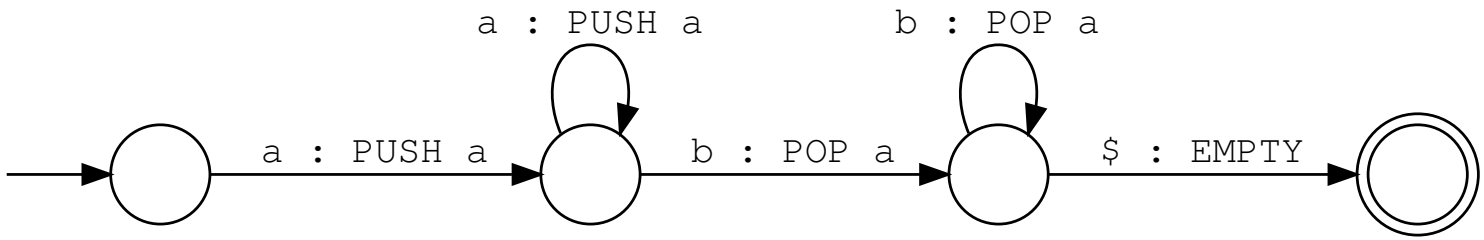
Notation:

- \$: The end of the input string.
- Edges decorated with "x : [PUSH/POP/PEEK] y", where x and y are tokens.

Example

A stack machine to recognize $\{a^n b^n : n \in \mathbb{N}\}$:



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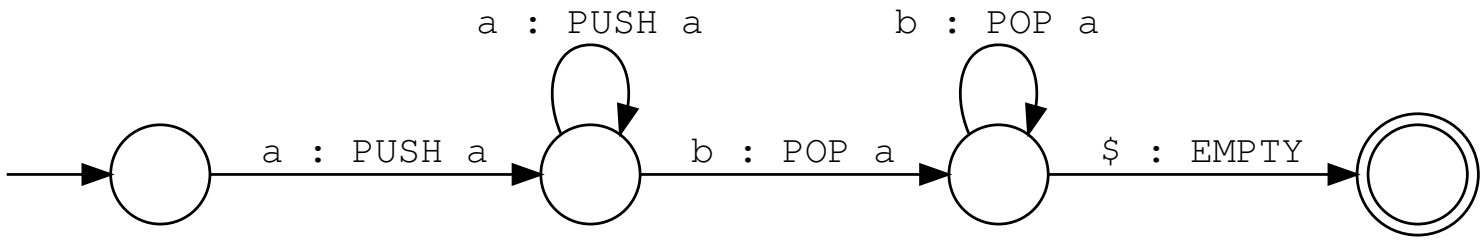

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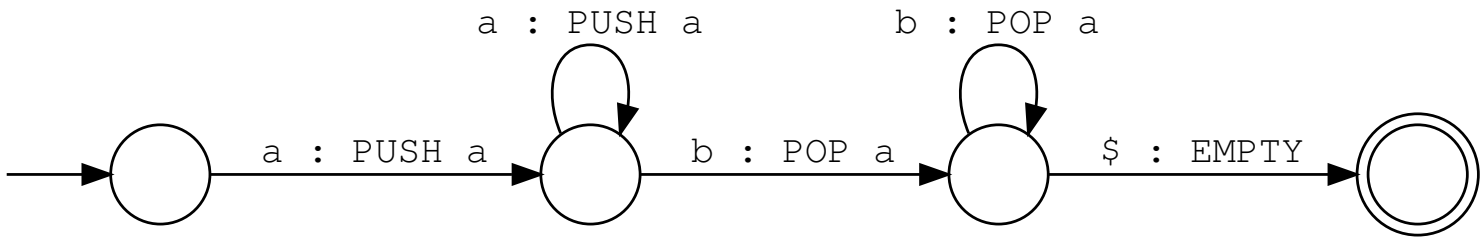

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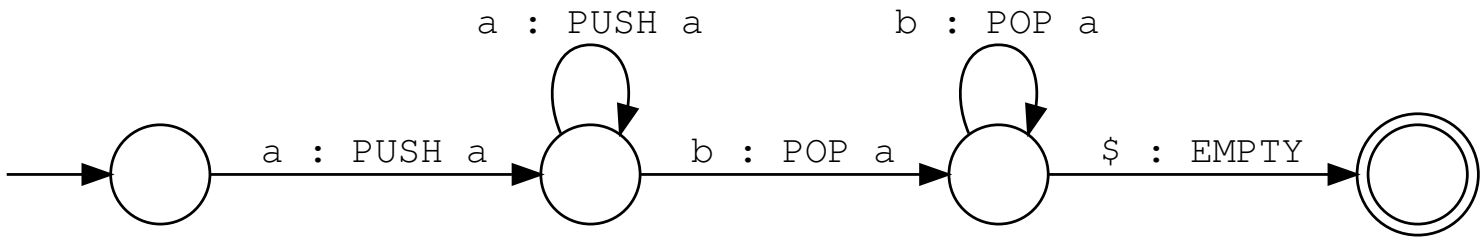

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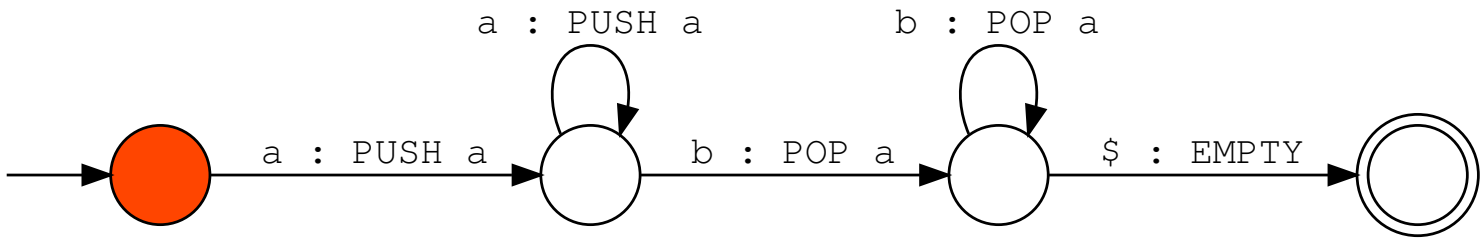
- Count up the number of a's.
- Count off the number of b's.

Example

Input: aabb$
Stack:



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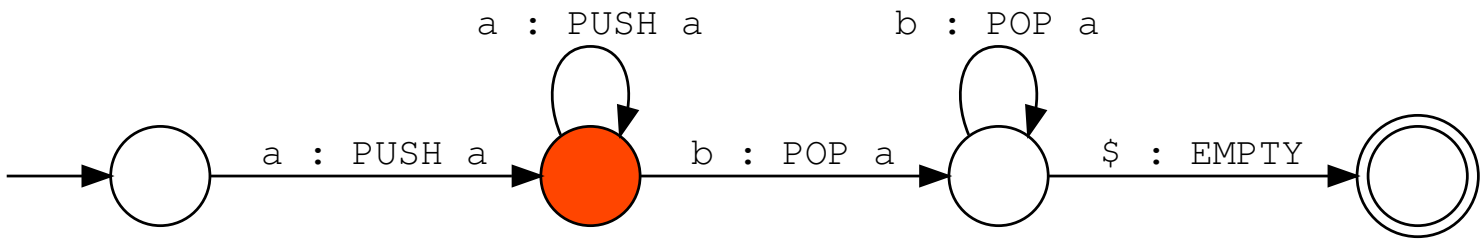
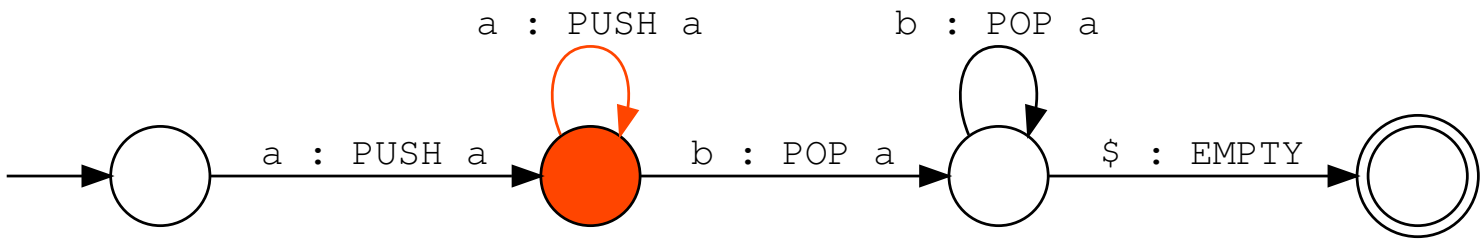


Example

Input: aabb$

Stack: aa



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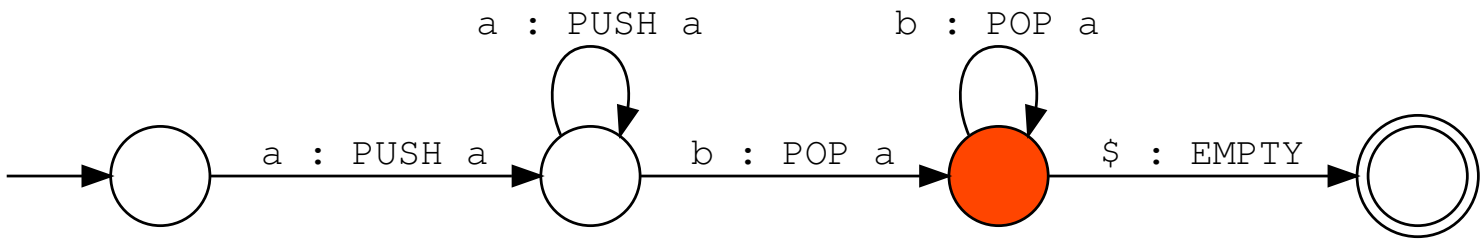

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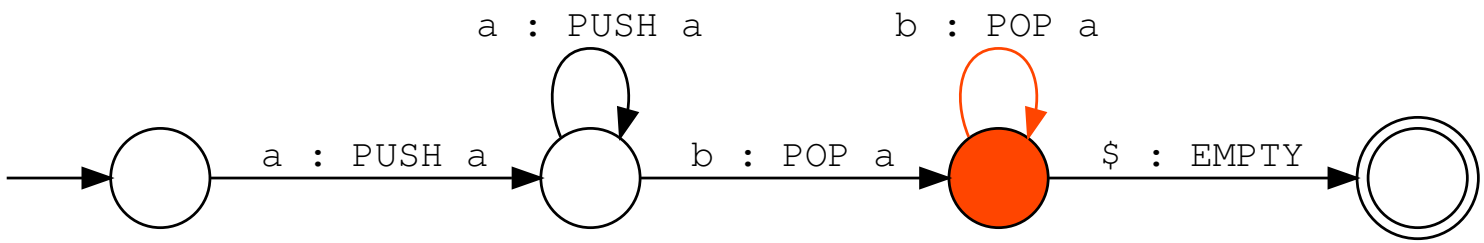

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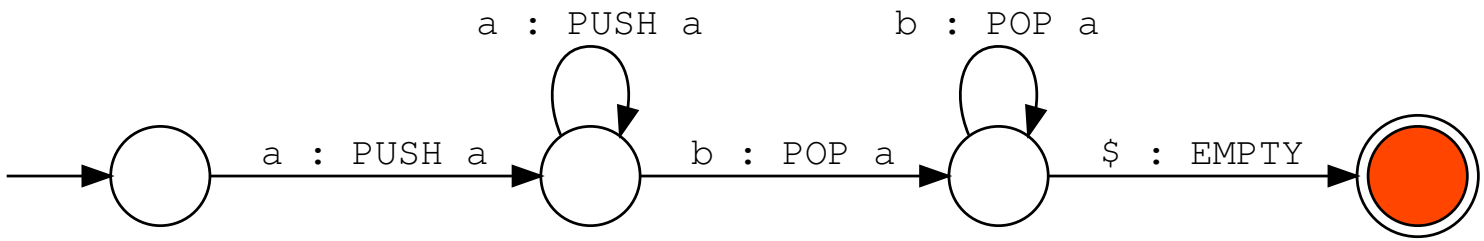
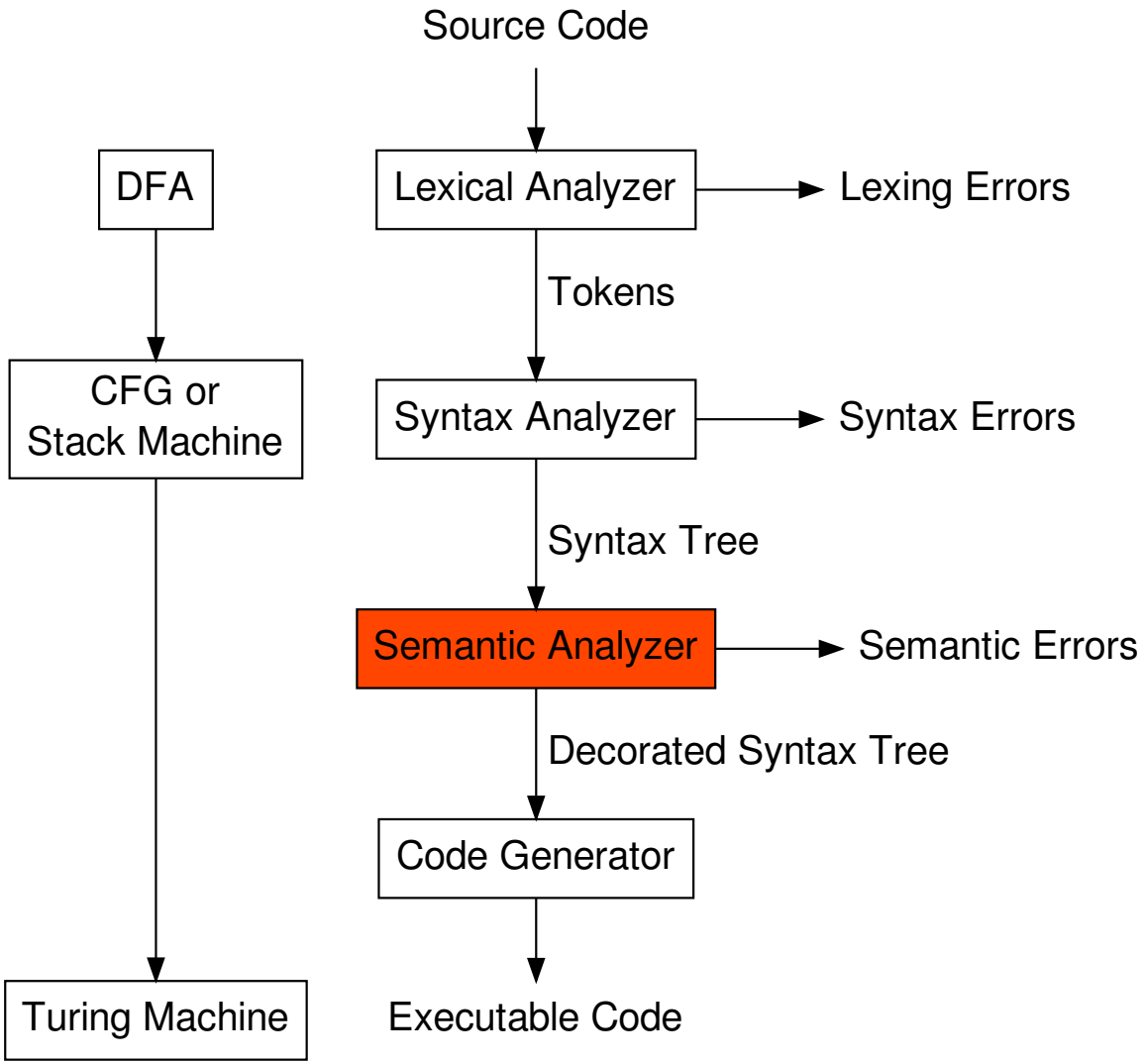


Semantic Analysis



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- Type — what a variable is supposed to represent.

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- Traverse the syntax tree:
  - Accumulate data from declarations.
  - Examine statements, making sure they're well formed.

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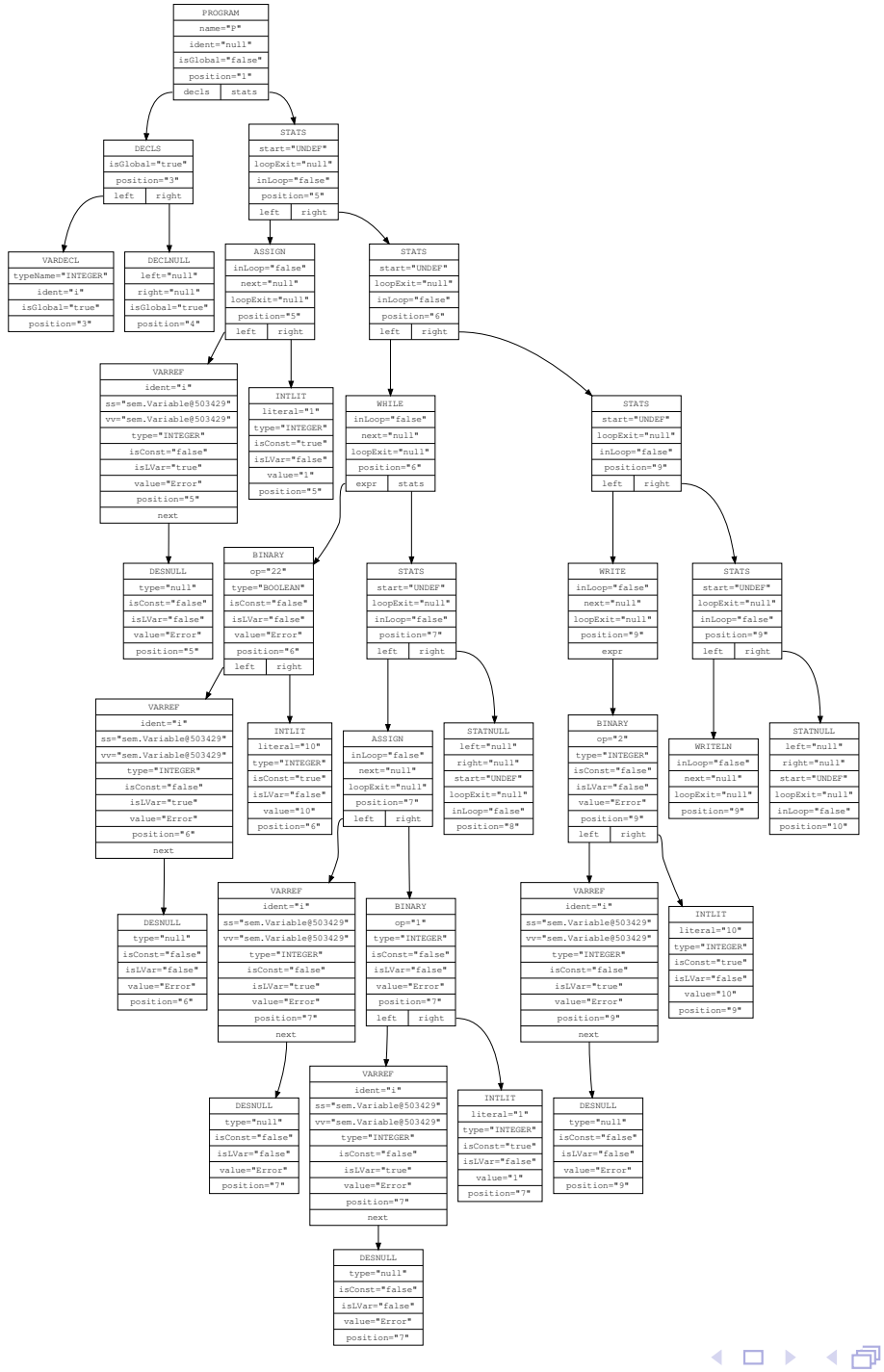
Important: The more you know at compile-time, the more efficient you can make your machine/byte code.


PROGRAM
name="p"
ident="null"
isGlobal="false"
position="1"
decls
stats
DECLS
isGlobal="true"
position="3"
left
right
STATS
start="UNDEF"
loopExit="null"
inLoop="false"
position="5"
left
right
VARDECL
typeName="INTEGER"
ident="i"
isGlobal="true"
position="3"
DECLNULL
left="null"
right="null"
isGlobal="true"
position="4"
ASSIGN
inLoop="false"
next="null"
loopExit="null"
position="5"
left
right
STATS
start="UNDEF"
loopExit="null"
inLoop="false"
position="6"
left
right
VARREF
ident="i"
ss="sem.Variable@503429"
vv="sem.Variable@503429"
type="INTEGER"
isConst="false"
isLVar="true"
value="Error"
position="5"
next
INTLIT
literal="1"
type="INTEGER"
isConst="true"
isLVar="false"
value="1"
position="5"
WHILE
inLoop="false"
next="null"
loopExit="null"
position="6"
expr
stats
STATS
start="UNDEF"
loopExit="null"
inLoop="false"
position="9"
left
right
DESNULL
type="null"
isConst="false"
isLVar="false"
value="Error"
position="5"
BINARY
op="22"
type="BOOLEAN"
isConst="false"
isLVar="false"
value="Error"
position="6"
left
right
STATS
start="UNDEF"
loopExit="null"
inLoop="false"
position="7"
left
right
WRITE
inLoop="false"
next="null"
loopExit="null"
position="9"
expr
STATS
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loopExit="null"
inLoop="false"
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next
INTLIT
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type="INTEGER"
isConst="true"
isLVar="false"
value="10"
position="6"
ASSIGN
inLoop="false"
next="null"
loopExit="null"
position="7"
left
right
STATNULL
left="null"
right="null"
start="UNDEF"
loopExit="null"
inLoop="false"
position="8"
BINARY
op="2"
type="INTEGER"
isConst="false"
isLVar="false"
value="Error"
position="9"
left
right
WRITELN
inLoop="false"
next="null"
loopExit="null"
position="9"
STATNULL
left="null"
right="null"
start="UNDEF"
loopExit="null"
inLoop="false"
position="10"
DESNULL
type="null"
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vv="sem.Variable@503429"
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value="Error"
position="7"
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BINARY
op="1"
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isConst="false"
isLVar="false"
value="Error"
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left
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value="Error"
position="9"
next
INTLIT
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isConst="true"
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isConst="false"
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isLVar="false"
value="1"
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DESNULL
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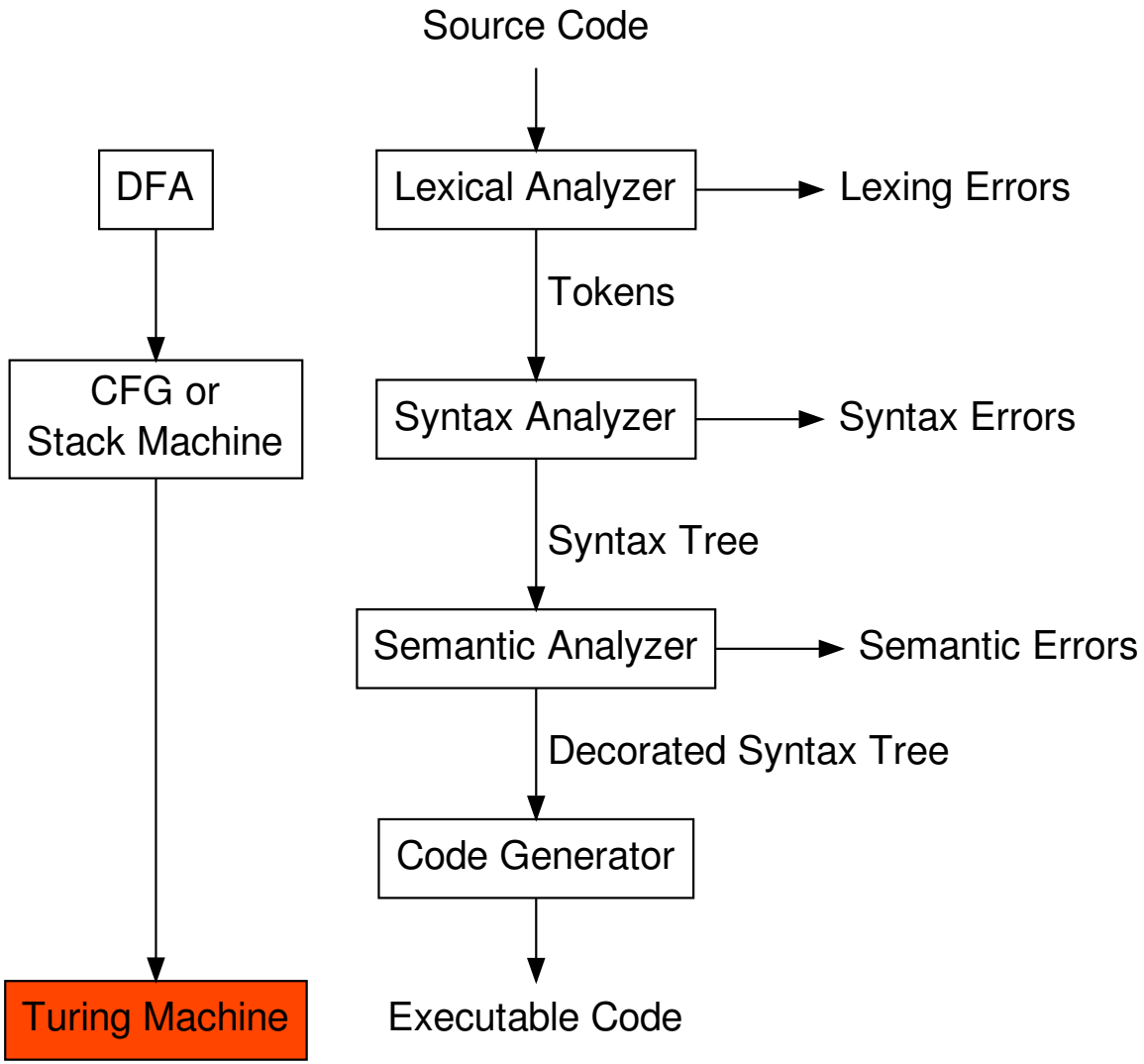
DESNULL
type="null"
isConst="false"
isLVar="false"
value="Error"
position="7"

Turing Machines



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  - Instructions for which way to move the read/write head.

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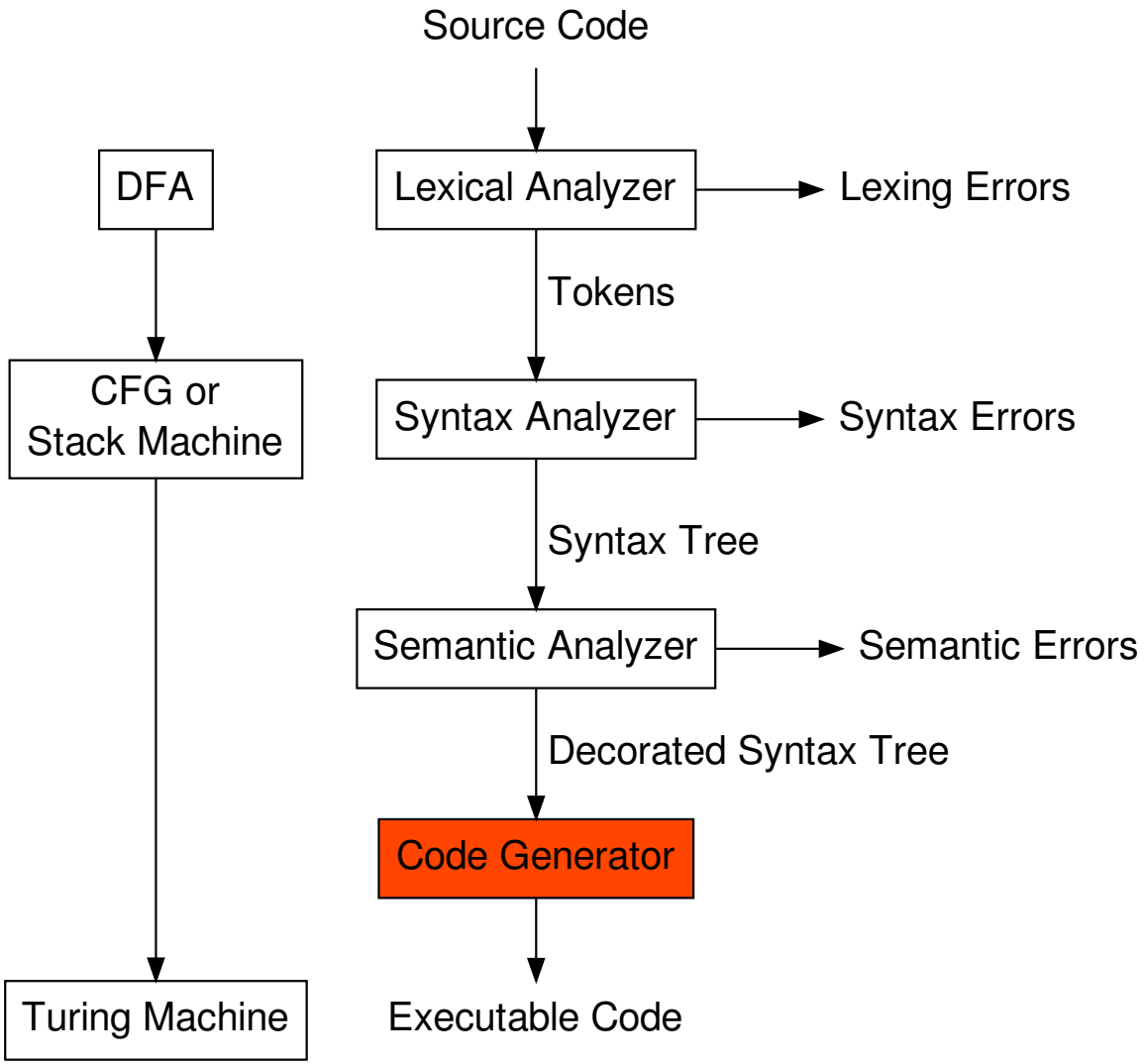
- Instead, we can pass through a number of intermediate languages.

Generating Machine Code



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  - Your compiler is tied to a particular architecture.

Example

```
PROGRAM FACTORIAL;

VAR i : INTEGER;
VAR prod : INTEGER;
BEGIN
    i := 1;
    prod := 1;
    WHILE i <= 10 DO
        prod := prod * i;
        i := i + 1;
    ENDDO;
    WRITE prod; WRITELN;
END.
```

Intermediate Representation Example

```
 1 | assign   v0  1
 2 | assign   v1  1
 3 | le       v2  v0  10
 4 | joz      v2  10
 5 | mul      v3  v1  v0
 6 | assign   v1  v3
 7 | add      v4  v0  1
 8 | assign   v0  v4
 9 | jump     3
10 | assign   v5  v1
11 | write    v5
12 | writeln
13 | end
```

Assembly Language Example (MIPS)

	Triple Code				MIPS Assembly			
1	assign	v0	1		li	$t0,	1	
					sw	$t0,	24($sp)	
2	assign	v1	1		li	$t0,	1	
					sw	$t0,	28($sp)	
3	le	v2	v0	10	lw	$t1,	24($sp)	
					li	$t2,	10	
					sle	$t0,	$t1,	$t2
					sw	$t0,	40($sp)	
4	joz	v2	10		label_3:			
					lw	$t0,	40($sp)	
					beq	$zero,	$t0,	label_10

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  - An array of memory (for storing variables).
  - A stack for performing calculations (in lieu of registers).

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Example: `5 * (3 + 2)` becomes

Instructions	Stack State
`push 5`	5
`push 3`	5, 3
`push 2`	5, 3, 2
`add`	5, (3+2)
`mul`	5(3+2)

Bytecode Example

```
 0 | alloc  2                    || 16 | get
 1 | push   0   % Addr.  i       || 17 | mul_i
 2 | push   1   % Value := 1     || 18 | store
 3 | store                       || 19 | push     0   % Addr.  i
 4 | push   1   % Addr.  prod    || 20 | push     0   % Addr.  i
 5 | push   1   % Value := 1     || 21 | get
 6 | store                       || 22 | push     1   % Value := 1
 7 | push   0   % WHILE start    || 23 | add_i
 8 | get                         || 24 | store
 9 | push   10  % Value := 10    || 25 | jump     7   % WHILE end
10 | le                          || 26 | push     1   % Addr.  prod
11 | joz    26                   || 27 | get
12 | push   1   % Addr.  prod    || 28 | write_I
13 | push   1   % Addr.  prod    || 29 | writeln
14 | get                         || 30 | end
15 | push   0   % Addr.  i       ||    |
```

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3. Decompilation : Given an executable E, can I generate source code S that compiles to E?